Measuring Availability Indexes With Small Samples for Component and Network Reliability Using the Sahinoglu–Libby Probability Model

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***Abstract*—With the advances in pervasive computing and wireless networks, the quantitative measurements of component and network availability have become a challenging task, especially in the event of often encountered insufficient failure and repair data. It is well recognized that the Forced Outage Ratio (FOR) of an embedded hardware component is defined as the failure rate divided by the sum of the failure and the repair rates; or FOR is the operating time divided by the total exposure time. However, it is also well documented that FOR is not a constant but is a random variable. The probability density function (pdf) of the FOR is the Sahinoglu–Libby (SL) probability model, named after the originators if certain underlying assumptions hold. The SL pdf is the generalized three-parameter Beta distribution (G3B). The failure and repair rates are taken to be the generalized Gamma variables where the corresponding shape and scale parameters, respectively, are not identical. The SL model is shown to default to that of a standard two-parameter Beta pdf when the shape parameters are identical. Decision Theoretic (Bayesian) solutions are employed to compute small-sample Bayesian estimators by using informative and noninformative priors for the component failure and repair rates with respect to three definitions of loss functions. These estimators for component availability are then propagated to calculate the network expected input–output or source–target (s–t) availability for four different fundamental networks given as examples. The proposed method is superior to using a deterministic way of estimating availability simply by dividing total up-time by exposure time. Various examples will illustrate the validity of this technique to avoid over- or underestimation of availability when only small samples or insufficient data exist for the historical lifecycles of components and networks.**

***Index Terms*—Bayes, beta, gamma, generalized three-parameter Beta distribution (G3B), informative, loss, Sahinoglu–Libby (SL), source–target (s–t) availability.**

NOMENCLATURE

FOR	Forced outage rate or unavailability index of a hardware or software component.
G3B	Generalized three-parameter beta RV.
MLE	Maximum likelihood estimate.
Q:Unavailability	RV for FOR, the probability that an item is inoperative at any point in time where q is a realization $q = 1 - r$.
R:Availability	Probability that an item is up (operating) at any point in time, where r is a realization. $r = 1 - q$.
RV	Random variable.
SL	Sahinoglu–Libby RV (same as G3B RV).
cdf	Cumulative probability density function of a given RV.
pdf	Probability density function of a given RV.
a	Number of occurrences of operative (up) times sampled.
b	Number of occurrences of debugging (down) times sampled.
c	Shape parameter of gamma prior for component failure rate λ.
d	Shape parameter of gamma prior for component recovery rate μ.
$E(q)$	Expected unavailability (= FOR) Estimator with informative prior using squared error loss.
$E(r)$	Expected availability $(= 1 - \text{FOR})$ estimator with an informative prior using squared error loss.
Q_{sys}	System unavailability random variable.
$\hat{q} = q_{\text{hat}}$	Estimator of RV q using a specified estimation method.
q^*	Expected unavailability (= FOR) estimator with informative prior using weighted squared error loss.
q^{**}	Expected unavailability (= FOR) estimator with noninformative prior when $\xi = \eta = 0, c = d = 1$ using weighted squared error loss.
$q^{**}_{\text{large-sample}}$	Unavailability (= FOR) large-sample asymptotic estimator of q^{**} if $a, b \rightarrow \infty$ where $(a/b) \approx 1$.
q_M	Median or Bayes estimator with informative prior for an absolute error loss function.
R_{sys}	System availability random variable.
$\sum_i$	Summation notation.

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π_i	Product notation.
$\hat{r} = r_{\text{hat}}$	Estimator of RV q using a specified estimation method.
r^*	Expected availability ($= 1 - $ FOR) estimator with informative prior using weighted squared error loss.
r^{**}	Expected availability ($= 1 - $ FOR) estimator with noninformative prior when $\xi = \eta = 0$, $c = d = 1$ using weighted squared error loss.
$r^{**}_{\text{large-sample}}$	Availability ($= 1 - $ FOR) large-sample asymptotic estimator of r^{**} if $a, b \to \infty$ where $(a/b) \approx 1$.
r_M	Median or Bayes estimator with informative prior for an absolute error loss function.
x_T	Total sampled up-time for a number of occurrences.
ξ	Inverse scale parameter of gamma prior for component failure rate λ.
y_T	Total sampled debugging (down) times for b number of occurrences of debugging activity.
η	Inverse scale parameter of gamma prior for component recovery rate μ.
mgf	Moment generating function.

I. Introduction and Motivation

In contrast to earlier research conducted by the main author on the reliability of software [19]–[22] and testing hardware [23], also cited by [24], this paper concentrates on the two main aspects: 1) Theory and 2) Application of the SL pdf to hardware components and networks [9]. It is assumed that embedded hardware elements (or firmware) are components in a computer network that have their failure and repair rates independently distributed with the two-parameter generalized Gamma(α_i, β_i) pdfs. The pdf of FOR $= \lambda/(\lambda + \mu)$, where λ = failure rate = number of failures/unit time and μ = repair rate = number of repairs/unit time. The ratio of $\lambda \sim$ Prior Gamma(α_1, β_1) to the sum of $\lambda \sim$ Prior Gamma(α_1, β_1) and $\mu \sim$ Prior Gamma(α_2, β_2), $\alpha_1 \neq \alpha_2$, $\beta_1 \neq \beta_2$, was documented by Sahinoglu and Libby in their respective published Ph.D. dissertations [3], [9], [10]. If the sampled historical up-times and down-times x_i and y_i, respectively, are exponentially distributed, then the reciprocal of the mean "up" or "down" times λ and μ have prior Gamma distributions on the grounds of the mathematical tractability and conjugacy property, and versatility in representing or approximating a wide range of distributions. Further, applying Bayesian inference techniques, *a posteriori* distributions of these rates are obtained following the merging of the prior information with the system's field data sampled. Subsequent treatment of the same problem can be found in Sahinoglu *et al.* [4], [5], Libby *et al.* [10], [11], and Pham-Gia *et al.* [13] in the form of G3B(α, β, L), where $L = \beta_1/\beta_2, \alpha = a+c, \beta = b+d$. This estimation problem was presented in a classical reference book on statistical distributions by Johnson *et al.*, where the SL (or G3B) was not yet existent in the 1970 edition, which only contained the default case of SL that was beta model [1, p. 182]. Later, the 1995 edition made a reference to the said pdf in the form of a G3B [2, p. 251]. The SL pdf appeared first in 1981 in the form of two independent Ph.D. dissertations [3], [10].

The pdf of availability for a multicomponent network of any complexity more than all series or parallel is simply infeasible to attain in a closed-form solution. Therefore, based on independent assumption of components, a function of a product of availabilities is necessary to estimate the desired network availability index (R_{sys}). Numerical examples on components and various sample network configurations are given in subsequent sections. The goal is to calculate the expected component and network availability accurately, despite the lack of large-sample historical data. This will be accomplished by calculating component availabilities, namely, Bayesian estimators, using selected informative and noninformative priors, and using three different loss (or penalty) functions.

II. SL Probability Model Formulation

In using the distribution function technique, the pdf of FOR $= q = \lambda/(\lambda + \mu)$ is obtained first by deriving its cdf $G_Q(q) = P(Q \leq q) = P((\lambda/(\lambda + \mu)) \leq q)$ and then taking derivative for $g_Q(q)$ as in Appendices (A.1)–(A.18), [3, pp. 26–32]–[5, p. 1487]

$$\begin{aligned} g_Q(q) &= \frac{\Gamma(a+b+c+d)}{\Gamma(a+c)\Gamma(b+d)} \\ &\quad \times \frac{(\xi+x_T)^{a+c}(\eta+y_T)^{b+d}(1-q)^{b+d-1}q^{a+c-1}}{[\eta+y_T+q(\xi+x_T-\eta-y_T)]^{a+b+c+d}} \quad (1) \\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-q)^{\beta-1}q^{\alpha-1} \\ &\quad \times \left\{ \beta_1^{\alpha}\beta_2^{\beta}\left[\frac{1}{\beta_2+q(\beta_1-\beta_2)}\right]^{\alpha+\beta} \right\} \\ &= \text{Beta}\ (\alpha,\beta) * (1-q)^{\beta-1}q^{\alpha-1} \\ &\quad * \left\{ \beta_1^{\alpha}\beta_2^{\beta}\left[\frac{1}{\beta_2+q(\beta_1-\beta_2)}\right]^{\alpha+\beta} \right\} \\ &= \text{Beta}\ (\alpha,\beta) * (1-q)^{\beta-1}q^{\alpha-1} \\ &\quad * \left[\frac{\beta_2}{\beta_2+q\beta_2(L-1)}\right]^{\alpha+\beta} L^{\alpha} \\ &= \text{Beta}\ (\alpha,\beta) * (1-q)^{\beta-1}q^{\alpha-1} \\ &\quad * \left[\frac{1}{1+q(L-1)}\right]^{\alpha+\beta} L^{\alpha} \quad (2) \end{aligned}$$

is the pdf of the RV $Q =$ FOR, where $\alpha = a + c$, $\beta = b + d$, $\beta_1 = \xi + x_T$ and $\beta_2 = \eta + y_T$, and $0 \leq q \leq 1$. If $L = \beta_1/\beta_2 = 1$ or $\beta_1 = \beta_2$, then the conventional two-parameter beta pdf is obtained.

On the other hand, an alternative original derivation of the same pdf under the *generalized multivariate beta distribution* is given in Libby [10, pp. 272–277]. This expression can also be reformulated in terms of SL($\alpha = a+c, \beta = b+d, L = \beta_1/\beta_2$) as follows:

$$g_Q(q) = \frac{L^{\alpha+c} q^{\alpha+c-1}(1-q)^{b+d-1}}{B(b+d, a+c).\,[1-(1-L)q]^{a+b+c+d}} \quad (3)$$

where

$$B(b+d, a+c) = \frac{\Gamma(a+c)\Gamma(b+d)}{\Gamma(a+b+c+d)} \quad \text{and} \quad L = \frac{(\xi + x_T)}{(\eta + y_T)}.$$

Note that if $L = 1$, SL pdf reduces to a Beta(α, β) pdf. Similarly, one obtains the pdf of the RV $R(\text{Availability}) = 1 - Q$, which by reparametrization of SL(α, β, L) into SL($\beta, \alpha L^{-1}$) leads to the following expression for the RV r

$$g_R(r) = \frac{L^{b+d}(1-r)^{a+c-1} r^{b+d-1}}{B(a+c, b+d).\,[1-(1-L)r]^{a+b+c+d}} \quad (4)$$

where

$$B(a+c, b+d) = \frac{\Gamma(a+c)\Gamma(b+d)}{\Gamma(a+b+c+d)} \quad \text{and} \quad L = \frac{(\eta + y_T)}{(\xi + x_T)}.$$

Densities of SL (or G3B) distributions have been cited by [13] for a variety of L (or λ per their nomenclature) values. From a strictly mathematical point of view, the presence of the parameter L allows SL pdf to take a variety of shapes besides the standard Beta(α, β) where $L = 1$. For example, when $\alpha = \beta$, the standard Beta(α, α) is symmetric with a mean at 0.5. However, the SL(α, α, L) distribution is not necessarily so, and can be positively or negatively skewed, depending on $L > 1$ and $L < 1$, respectively, because the mode, skewness, and kurtosis of SL RV now also depend on L. For $0 < L < 1$, the SL pdf stays below that of the corresponding standard Beta near zero but crosses the latter to become the greater of the two pdfs at $y_0 = \{1 - L^{\alpha_1/(\alpha_1+\alpha_2)}\}^{-1} - (1-L)^{-1}$ [2]. The reverse action holds true for $L > 1$ with the same crossing point y_0. The desired moments as well as median and quartiles have been generated through the use of a Java code by the corresponding author. These values will be listed in the example in Table I.

The major drawback to the distribution is that there is no closed form for finite estimates of the moments. The moment generating function for the univariate SL distribution is an infinite series [11]. This is why Bayes estimators can be of practical use. The Bayes estimators, in closed or numerically integrable forms, are derived next. Simpson's trapezoidal formula is used for conducting the numerical integration to compute these moments as shown in Table I.

III. Bayes Estimators for Various Informative Priors and Loss Functions

Various studies have substantiated that the finite moments do not exist in closed form for the SL(α, β, L). Standard methods only lead to unfavorable recursive solutions, a situation that poses a dead-end as in [11] and [13]. However, an alternative way of finding some meaningful and computable Bayes estimates for the unavailability RV Q and availability $R = 1 - Q$ is achieved by using Bayes estimation techniques with various loss functions [3]. Two popularly used squared error loss functions and one absolute error loss function will be examined as penalty functions.

A. Squared Error Loss Function

Let q_{hat} denote an estimate of the RV denoted to be $Q \equiv$ FOR. The loss incurred, $L(q, q_{\text{hat}})$, in estimating the true but unknown q can be defined at will. Usually, the loss penalty increases as the difference between q and q_{hat} increases. Therefore, the squared error loss function $L(q, q_{\text{hat}}) = (q - q_{\text{hat}})^2$ has found favor where the risk of taking a decision is

$$\text{Risk}(q, q_{\text{hat}}) = E\{L(q, q_{\text{hat}})\} = E(q - q_{\text{hat}})^2. \quad (5)$$

This would then be the variance of the estimator Q, penalizing larger differences more in classical least squares theory as in [14] and [15]. Bayes estimator of q in our problem with respect to squared error loss function is the first moment or expected value of the RV q using its pdf [3], [9]. This follows from the fact that $E(q - q_{\text{hat}})^2$, if it exists, is a minimum when $q_{\text{hat}} = E(q)$, i.e., the mean of the conditional (posterior) distribution of q given $\underline{x}$ (up-times) and $\underline{y}$ (down-times). Then $E(q)$ is the Bayes solution

$$E(q) = E_Q[q|\underline{X} = \underline{x}, \underline{Y} = \underline{y}] = \int_0^1 q g_Q(q) dq. \quad (6)$$

Similarly, the Bayes estimator of r, i.e., r_{hat} with respect to squared error loss function using informative prior is the first moment or expected value of the RV r using its pdf in (3). That is

$$E(r) = E_R[r|\underline{X} = \underline{x}, \underline{Y} = \underline{y}] = \int_0^1 r g_R(r) dr = 1 - E(r). \quad (7)$$

B. Absolute Error Loss Function

Similarly by Hogg and Craig [15, p. 262], the median of the RV Q is the Bayes estimator using an informative prior when the loss function is given as $L(q, q_{\text{hat}}) = |q - q_{\text{hat}}|$. If $E(|q - q_{\text{hat}}|)$ exists, then $q_{\text{hat}} = q_{.5}$ minimizes the loss function, i.e., the median of the conditional (posterior) distribution of q given $\underline{x}$ (up-times) and $\underline{y}$ (down-times). Median is very resistant to changes. Then $q_{.5}$ or median of q, q_M is the Bayes solution.

That is, q_M is the 50th percentile or .5 quantile, or second quartile for q, as it follows

$$0.5 = \int_0^{q_{.5}} g_Q(q) dq. \quad (8)$$

Similarly, $r_M = 1 - q_M$, is the 50th percentile or .5 quantile, or second quartile for r, as it follows:

$$0.5 = \int_0^{r_{.5}} g_R(r) dq. \quad (9)$$

C. Weighted Squared Error Loss Function

Weighted squared error loss is of considerable interest to statisticians and engineers [16], and has the attractive feature of allowing the squared error to be weighed by $w(q)$, which is a function of q. This will reflect that a given error of estimation often varies in penalty according to the value of q. Then, the weighted squared error loss function selected in such cases is as follows:

$$L(q, q_{\text{hat}})^2 = \frac{C(q - q_{hat})^2}{q(1-q)} = w(q)(q - q_{\text{hat}})^2. \quad (10)$$

With this loss function, the Bayes estimator of q is given as follows (see (B.1)–(B.7) for details):

$$\begin{aligned} q^* &= \frac{\int_Q q w(q) h(q|\underline{X} = \underline{x}, \underline{Y} = \underline{y}) dq}{\int_Q w(q) h(q|\underline{X} = \underline{x}, \underline{Y} = \underline{y}) dq} \\ &= \frac{E_Q\left[q w(q)|\underline{X} = \underline{x}, \underline{Y} = \underline{y}\right]}{E_Q\left[w(q)|\underline{X} = \underline{x}, \underline{Y} = \underline{y}\right]}. \end{aligned} \quad (11)$$

Utilizing (B.7) in Appendix B, and by assuming the coefficient of the weight function $w(q)$, $C = 1$

$$q^* = \frac{E_Q\left[q \frac{1}{q(1-q)}\right]}{E_Q\left[\frac{1}{q(1-q)}\right]} = \frac{E_Q\left[(1-q)^{-1}\right]}{E_Q\left[q^{-1}(1-q)^{-1}\right]} \quad (12)$$

where $(1-q)^{-1} = (1/(1-(\lambda/(\lambda+\mu)))) = 1 + (\lambda/\mu)$ and $q^{-1}(1-q)^{-1} = (1+(\mu/\lambda))(1+(\lambda/\mu)) = (1+(\mu/\lambda)+(\lambda/\mu)+1) = 2+(\mu/\lambda)+(\lambda/\mu)$. This gives, when substituted into (12), and using posterior gamma distributions by (A.7) and (A.8)

$$q^* = \frac{\int_{\lambda=0}^{\infty}\int_{\mu=0}^{\infty}\left(1+\frac{\lambda}{\mu}\right) h_1(\lambda|\underline{X} = \underline{x}) h_2(\underline{Y} = \underline{y}) d\lambda d\mu}{\int_{\lambda=0}^{\infty}\int_{\mu=0}^{\infty}\left(2+\frac{\lambda}{\mu}+\frac{\mu}{\lambda}\right) h_1(\lambda|\underline{X} = \underline{x}) h_2(\mu|\underline{Y} = \underline{y}) d\lambda d\mu}. \quad (13)$$

Since $h_1(\lambda|\underline{x}) = (1/\Gamma(a+c))(x_T+\xi)\lambda^{a+c-1}\exp\{-\lambda(x_T+\xi)\}$ is the Gamma$\{a+c, (x_T+\xi)^{-1}\}$, and $h_2(\mu|\underline{y}) = (1/\Gamma(b+d))(y_T+\eta)\mu^{a+c-1}\exp\{-\mu(y_T+\eta)\}$ is the Gamma$\{b+d, (y_T+\eta)^{-1}\}$, then

$$\iint_{\lambda\ \mu} h_1(\lambda|\underline{x}) h_2(\mu|\underline{y}) d\lambda d\mu = 1.0 \quad (14)$$

$$\iint_{\lambda\ \mu} \lambda h_1(\lambda|\underline{x}) \mu^{-1} h_2(\mu|\underline{y}) d\lambda d\mu = \left(\frac{a+c}{\xi+x_T}\right)\left(\frac{\eta+y_T}{b+d-1}\right) \quad (15)$$

where the expectation of an RV distributed with Gamma(α, β) is $\alpha\,\beta$. Therefore $E(\lambda) = (a+c)/(\xi+x_T)$ using (A.7). Using (A.8), the expectation of the reciprocal of a RV distributed with Gamma(α, β) is $1/\beta(\alpha-1)$ as follows:

$$E\left(\frac{1}{\mu}\right) = \int_{\mu} \frac{1}{\mu} h_2(\mu|\underline{y}) d\mu = \left(\frac{\eta+y_T}{b+d-1}\right). \quad (16)$$

Similarly, employing the same "expectation of the reciprocal of a Gamma RV" principle, and by (A.7)

$$E\left(\frac{1}{\lambda}\right) = \int_{\lambda} \frac{1}{\lambda} h_1(\lambda|\underline{x}) d\lambda = \left(\frac{\xi+x_T}{a+c-1}\right). \quad (17)$$

Now, putting it all together as dictated by (12)

$$q^* = \frac{1 + \frac{(a+c)(\eta+y_T)}{(\xi+x^T)(b+d-1)}}{2 + \frac{(b+d)(\xi+x_T)}{(\eta+y_T)(a+c-1)} + \frac{(a+c)(\eta+y_T)}{(\xi+x_T)(b+d-1)}} \quad (18)$$

is the small sample (before the sampled sums x_T and y_T predominate) Bayes estimator with respect to a weighted squared error loss function as given above and suggested for use in the conventional studies to stress for tail values, such as $q = 0.1$ or $q = 0.9$, where the value of the weight function increases. This is a small-sample estimator as opposed to that of asymptotic requiring large-sample data, thereby reflecting insufficient unit history. Here, $w(q)$ was conveniently taken to be $[q(1-q)]^{-1}$. For the special case when placing $\xi = \eta = 0$ (that is, scale parameters are infinite), $c = d = 1$ in (18) for noninformative (flat) priors, q^* becomes

$$\begin{aligned} q^{**} &= \frac{1 + \frac{(a+1)(y_T)}{(x_T)(b)}}{2 + \frac{(b+1)(x_T)}{(y_T)(a)} + \frac{(a+1)(y_T)}{(x_T)(b)}} \\ &= \frac{x_T y_T ab + y_T^2 a(a+1)}{2x_T y_T ab + y_T^2 a(a+1) + x_T^2 b(b+1)}. \end{aligned} \quad (19)$$

Finally, q^{**} asymptotically approaches the $q_{\text{large-sample}}$ estimator, the same as that of the MLE obtained by conventional (non-Bayesian) methods, which occurs when the influence of *a priori* parameters vanishes. This happens when the observed number of samples $a, b \to \infty$ in (19) such that $a+1 \approx b$ and $b+1 \approx a$. Then, (19) will reduce to (18)

$$\begin{aligned} q^{**}_{\text{large-sample}} &= \frac{1 + \frac{y_T}{x_T}}{2 + \frac{x_T}{y_T} + \frac{y_T}{x_T}} \\ &= \frac{x_T + y_T}{2(y_T x_T) + (y_T)^2 + (x_T)^2} \frac{x_T y_T}{x_T} \\ &= \frac{(x_T + y_T) y_T}{(x_T + y_T)^2} \\ &= \frac{y_T}{x_T + y_T}. \end{aligned} \quad (20)$$

By a similar process, we can reparametrize for the RV $r = 1 - q$ as in (18). This reparameterization is achieved since, if $Q \sim$ SL(α, β, L), then its complement $R = (1 - Q)$ is SL(β, α, L^{-1}), a characteristic that is similar to the one employed for the standard Beta(α, β) as in (4). Note that $E(r) = 1 - E(q)$. Then

$$r^* = \frac{1 + \frac{(\xi+x^T)(b+d)}{(a+c-1)(\eta+y_T)}}{2 + \frac{(b+d)(\xi+x_T)}{(\eta+y_T)(a+c-1)} + \frac{(a+c)(\eta+y_T)}{(\xi+x_T)(b+d-1)}} = 1 - q^* \quad (21)$$

is the Bayes estimator of the availability $R = 1-Q$, with respect to a weighted squared error loss. Here, $w(r)$ was similarly taken for (10) to be $[r(1-r)]^{-1}$. For the special case when $\xi = \eta = 0$,

TABLE I
INPUT AND OUTPUT TABLE FOR COMPONENT AND NETWORK APPLICATIONS

Input Data	***Component 1***	***Component 2***	***Component 3***	***Component 4***
a(#failure events)	10	5	10	100
b (#repair events)	10	5	10	100
$\mathbf{x_T}$ (failure total)	1000hrs	25hrs	1000hrs	10000hrs
$\mathbf{y_T}$ (repair total)	111.11hrs	5hrs	111.11hrs	1111.1hrs
c (shape for λ)	0.02	0.2	0.5	0.5
ξ (inverse scale)	1	1	1	1
d (shape for μ)	0.1	2	2	2
η (inverse scale)	1	0.5	.25	.25
Case1: Single	***Component 1*** *(Screenshot 3)*	***Component 2***	***Component 3***	***Component 4***
r *	0.907325	0.882397	0.917820	0.902027
r **	0.906655	0.849595	0.906655	0.900715
E(r)=Mean	0.890983 *(Screenshot 1)*	0.758064	0.879397	0.897921
r_M=Median	0.898534	0.77545	0.88683	0.89865
$r^{**}_{\text{large sample}}$	0.9	0.8333	0.9	0.9
Std. Deviation (= √Bayes Risk)	.045=√.00203	.107=√.0115	.046=√.0021	.013=√.00017
IQR(Inter-quartile Range)	0.06 *(Screenshot 2)*	0.14	0.06	0.015
Skewness	-1.11	-0.901	-1.04	-0.339
Kurtosis	2.11	0.985	1.846	0.2
Case2: System w/ Identical Comp.1	**Sys.Config.I (Series)** *(Screenshot 4)*	**Sys. Config.II (Parallel)**	**Sys.Config.III (Series-in-Parallel)**	**Sys.Config.IV (Parallel-in-Series)**
R*	0.677723	0.999926	0.968721	0.907495
R**	0.675723	0.999924	0.968324	0.906723
E(R)	0.633723	0.999858	0.958395	0.897612
R_M	0.651836	0.999919	0.967248	0.982031
$R^{**}_{\text{large sample}}$	0.656103	0.999900	0.963900	0.980100
Case3: System w/ Different Comps *(Screenshot 5)*	**Sys.Config.I (Series)**	**Sys.Config.II (Parallel)**	**Sys.Config.III (Series-in-Parallel)**	**Sys.Config.IV (Parallel-in-Series)**
R*	0.668156	0.999920	0.967982	0.981851
R**	0.622128	0.999846	0.956336	0.976621
E(R)	0.533335	0.999675	0.931719	0.961639
R_M	0.555289	0.999782	0.942783	0.968868
$R^{**}_{\text{large sample}}$	0.607502	0.999833	0.952501	0.973500

Sample input parameters and estimators for Case 1) of single components and for Cases 2) and 3) of system configurations A to D as in Fig. 1. Case 1) single components (nonsystem), Case 2) system study with all identical components, Case 3) system study with all nonidentical components using 1 to 4 in a sequence as needed. Gamma prior parameters are selected from sample plots in Fig. 2 to show degrees of skewness. For example, $d(\text{shape}) = 2$, $\eta(\text{inverse scale}) = .5$ is almost symmetric, $c(\text{shape}) = .5$, $\xi(\text{inverse scale}) = 1$ is a hyperexponential. The scale parameters in Fig. 2 are the reciprocals of the inverse scales in this paper.

$c = d = 1$, i.e., for noninformative or flat priors, r^* becomes r^{**} as in (22)

$$r^{**} = \frac{1 + \frac{(x_T)(b+1)}{(a)(y_T)}}{2 + \frac{(b+1)(x_T)}{(y_T)(a)} + \frac{(a+1)(y_T)}{(x_T)(b)}} = 1 - q^{**}. \quad (22)$$

If the sample sizes of up- and down-times a and b are too large such that $(a/b) \to 1$, then similarly, r^{**} approaches the $r_{\text{large-sample}}$ as $a, b \to \infty$ as follows:

$$\begin{aligned} r^{**}_{\text{large-sample}} &= \frac{1 + \frac{x_T}{y_T}}{2 + \frac{x_T}{y_T} + \frac{y_T}{x_T}} \\ &= \frac{x_T + y_T}{2(y_T x_T) + (y_T)^2 + (x_T)^2} \frac{x_T y_T}{y_T} \\ &= \frac{(x_T + y_T)x_T}{(x_T + y_T)^2} \\ &= \frac{x_T}{x_T + y_T}. \end{aligned} \quad (23)$$

IV. AVAILABILITY CALCULATIONS FOR SIMPLE SERIES-PARALLEL NETWORKS

Four different fundamental topologies from easiest to hardest will be studied. Therefore, in evaluating various network availability or unavailability, exact values are used such as πq_i or πr_i, where $r_i = 1 - q_i$.

I) *Series Systems:* $R_{\text{sys}} = \pi_1^m r_i$ and $Q_{\text{sys}} = 1 - R_{\text{sys}}$, where π denotes "the product of" and $m = \#$series subsystems.

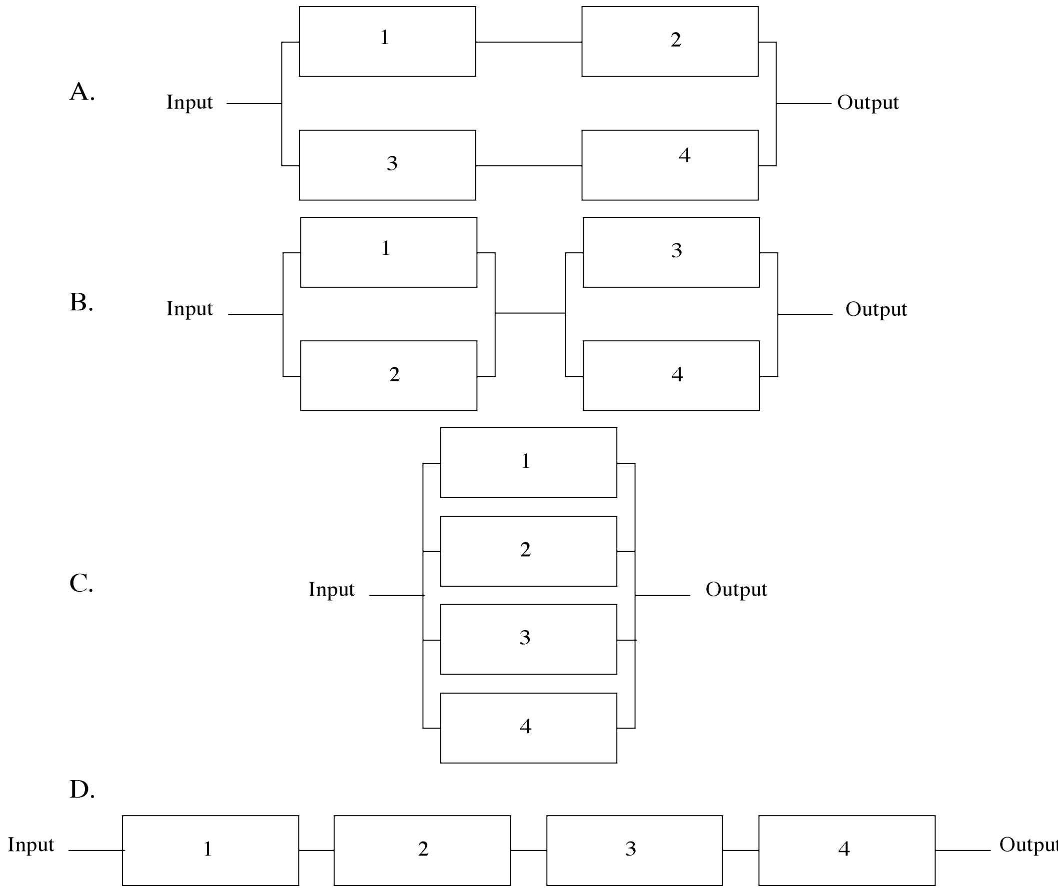

Fig. 1. A: Series-in-parallel (III), B: parallel-in-series (IV), C: parallel (II), and D: series (I).

II) *Active Parallel Systems:* $Q_{\text{sys}} = \pi_1^n q_i$ and $R_{\text{sys}} = 1 - Q_{\text{sys}}$, where $n = \#$parallel paths.

III) *Series-in-Active Parallel:* $Q_{\text{sys}} = \pi_1^n[(1 - \pi_1^m r_i)]$ and $R_{\text{sys}} = 1 - Q_{\text{sys}}$.

IV) *Active Parallel-in-Series:* $R_{\text{sys}} = \pi_1^m[(1 - \pi_1^n q_i)]$ and $Q_{\text{sys}} = 1 - R_{\text{sys}}$.

Three cases are tested and illustrated [6]. See Table I to observe the differences between the Bayesian estimators [7], [8]. Observe also input data and results for Cases 1), 2), and 3) in Table I. In a coding algorithm in the Java program specifically written for this purpose, postfix $(*)$ is used for denoting series and postfix $(+)$ is used for denoting active parallel systems [17, p. 454]. The components are treated at most two at a time [18, pp. 298–299]. Here are some examples for the four different fundamental series-parallel networks. Using a hand calculator, for all $r_i = 0.9$, $R_{\text{sys}}(\mathbf{I}) = .6561$, $R_{\text{sys}}(\mathbf{II}) = .9999$, $R_{\text{sys}}(\mathbf{III}) = .9639$, $R_{\text{sys}}(\mathbf{IV}) = .9801$. Let us code each configuration and code them using postfixes.

I) 1,2,$*$, 3,4, $*$, $*$ denotes all four components in series. For Case 3) of nonidentical components from 1 to 4, $R_{\text{sys}} = r_1 r_2 r_3 r_4$. For Case 2), let all r_is be identical.

II) 1,2,+, 3,4, +, + denotes all four components in active parallel. For Case 3) with nonidentical components from 1 to 4, $R_{\text{sys}} = 1 - q_1 q_2 q_3 q_4$. For Case 2), let all q_is be identical.

III) 1,2,$*$, 3,4, $*$, + denotes that the two components (1 and 2) first in series are in active parallel with the two other components (3 and 4) in series. $R_{\text{sys}} = 1 - \{(1 - r_1 r_2)(1 - r_3 r_4)\}$.

IV) 1,2,+, 3,4, +, $*$ denotes that the two components (1 and 2) first in active parallel are in series with the two other components (3 and 4) that are in active parallel. $R_{\text{sys}} = (1 - q_1 q_2)(1 - q_3 q_4)$.

V. Discussions and Conclusion

This paper concentrates on the two main aspects: 1) theory and 2) application of the SL pdf to hardware components and networks [9]. In the theory section of this paper, a detailed derivation of the univariate SL(α, β, L) pdf as originally noted in Sahinoglu and Libby's Ph.D. dissertations is presented with reference to a Bayesian process for informative and noninformative priors using absolute, squared, and weighted squared error loss functions. Therefore, SL(α, β, L) pdf is the continuous probability function of the RV of unavailability or availability of a component in a network whose lifetime can be decomposed into operating (up) and nonoperating (down) periods in a dichotomous setting. Up- and down-times are general Gamma models where both shape and scale parameters are different from each other. Beta(α, β) is a special case of the SL(α, β, L) where the ratio of the respective Gamma shape parameters for the failure and repair rates are identical, $L = 1$. Further, difficulties in calculating the closed-form moments of the said RV are outlined in this paper, therefore suggesting

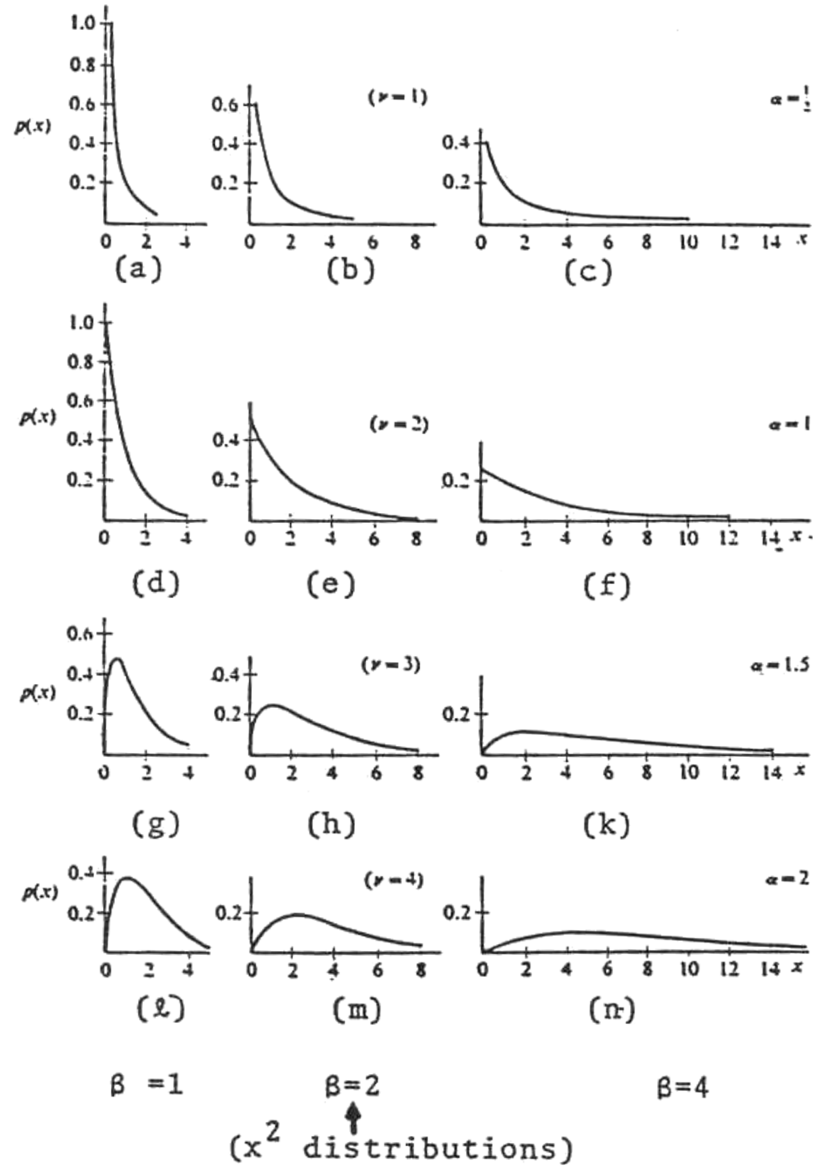

Fig. 2. Gamma density functions (α = shape parameter and β = scale parameter).

Bayesian estimators using different informative priors with respect to various meaningful loss functions.

In the application section of this paper, the reader is primarily referred to Table I and Figs. 1 and 2 for input parameters and output estimators of availability for the four different components selected as examples. Case 1) is for single components only, without network consideration. Case 2) is for networks of different topology, with Configuration 1 (series), Configuration 2 (parallel), Configuration 3 (series-in-parallel), and Configuration 4 (parallel-in-series), when component 1 input data are used invariably for all four components that make up the configuration. Case 3) is the same as Case 2) except for the detail that the components are not identical and selected in the order from component 1 to 4 as listed in Table I.

The variances of both q and r are identical as expected, and so are their standard deviations. r is left-skewed with a negative sign, and q is right-skewed with a positive sign at a mirror image. Standard deviations for both are 0.045, skewness yield -1.11 and 1.11, respectively, and data-resistant medians are 0.8985 and 0.1015, respectively, all for Component 1. Both RVs have positive kurtosis $(= 2.11)$, which denotes that these RV have leptokurtic distributions where the tail thickness is above that of a standard normal distribution. Moreover, the kurtosis is above 1.0, indicating a thicker tail than that of the standard normal distribution with a reference of unity. Simpson's trapezoidal rule is used with a very fine precision to obtain these results on the moments of r and q, $P(0 < r < 1) = 1$, or $P(0 < q < 1) = 1$ for intervals like $P(0.5 < r < 0.9) = 0.514$ or $P(0 < r < 0.8985) = 0.5$, or $P(0 < q < .1015) = 0.5$ as some examples.

In the upper input part of Table I, complying with the given definitions, the gamma priors for the failure and repair rates as to indicating left or right skewness or symmetry can be chosen by the analyst at will with an educated guess or expert judgment as in Fig. 2 [1]. For example, prior inputs of the failure and repair rates for component 3 in Table I with $c = 0.5$ and $\xi = 1$ denote a peaked hyperexponential as on the very upper left in Fig. 2,

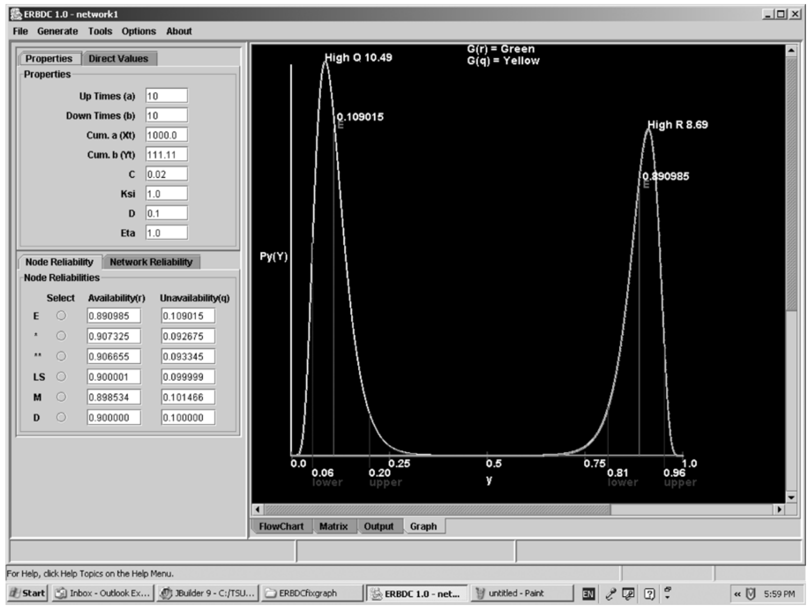

Fig.3. Ninety percent confidence intervals for the right skewed unavailability (on the left) and the left skewed availability (on the right) RVs. (See Table I.)

whereas $d = 2, \eta = 0.25$ resembles an almost flattened-mound-like shape at most rising to a probability of 0.1 given in the very bottom right.

Keeping all other parameters constant, it may be observed that as the *a priori* distributions of the failure and repair rates are more realistic, i.e., as the sampled up and down time sizes (number of occurrences)a and b get larger and larger and correspondingly the total up- and down-times increase, then the mean of the RV $q(= \text{FOR})$ approaches the MLE used by conventional methods [5]. It may be observed for component 4 in Table I, for example, that by taking a large number of failure and repair events, such as 10 000, the small-sample Bayesian estimators converge to the large-sample estimator of x_T divided by the sum of y_T and x_T, which is 0.9. When the total up- and down-times for component 4 were elevated to $x_T = 100\,000$ and $y_T = 11\,111.1$ for $a = 100$ and $b = 100$, the Bayesian estimators $E(r) = 0.897\,930$, $r^* = 0.893\,939$, and $r^{**} = 0.892\,685$ all became closer and closer to the 0.9 mark, the large-sample estimate, or the conventional MLE. Further supporting this fact in a sequence of sensitivity analyses: $a = 150$, $b = 150$, $x_T = 1\,500\,000$, and $y_T = 166\,666.65$ yielded $E(r) = 0.898\,621$, $r^* = 0.895\,967$, $r^{**} = 0.895\,117$; and $a = 170$, $b = 170$, $x_T = 1\,700\,000$, $y_T = 188\,888.87$ resulted in $E(r) = 0.898\,783$, $r^* = 0.896\,443$, $r^{**} = 0.895\,684$, each time converging to the conventional MLE.

In the event of insufficient data, it is demonstrated that the Bayes estimators depending on the type of priors and penalty functions are good alternatives when large sample asymptotic estimators cannot be obtained. A wise choice of prior parameters and penalty functions is an important requirement, since the more realistic these judgments are, the more accurate the results will be. Otherwise, assuming large sample estimators when large sample data is not available may lead to erroneous calculations of component and propagated network availability. Therefore, in an algorithmic sequence, do the following.

1) Decide on your prior functions for your components by considering a list of gamma plots such as in Fig. 2 for your failure and repair rates as shown in Table I.
2) Decide on your loss or penalty function.
3) Decide on whether to use informative or noninformative priors.
4) According to which decisions are made, choose your Bayesian estimator(s) such as r^*, r^{**}, $E(r)$, r_M.

These rules then hold also for the network applications, namely, R^*, R^{**}, $E(R)$, R_M according to a given topology, a sample of which is in Fig. 1. Moreover, these calculations are applicable to any complex (nonseries-parallel) networks, which have not been illustrated due to lack of space [25], [26].

Finally, Fig. 3 illustrates some further applications as follows:

1) 90% confidence intervals for the population mean using the Bayes Estimator for a single component regarding unavailability (q) and availability (r) RV for component 1 data in Table I;
2) medians and interquartile ranges for a single component regarding unavailability (q) and availability (r) RV, using component 1 data in Table I;
3) comparison of various availability estimators for a single component, using component 1 data in Table I;
4) comparison of various availability estimators for a network with four components in series, using component 1 data in Table I.
5) density plots for the right skewed unavailability and the left skewed availability RV side by side using four different component data as in Table I.

APPENDIX A

The results shown in many textbooks indicate that the residence times in the down-state prior to the up-state, or vice versa,

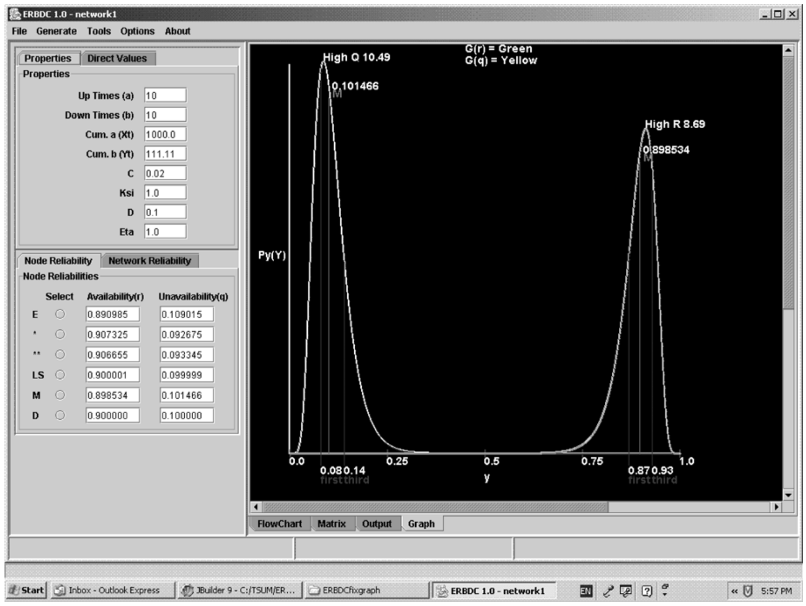

Fig.4. Medians and interquartile ranges for unavailability and availability RVs. (See Table I.)

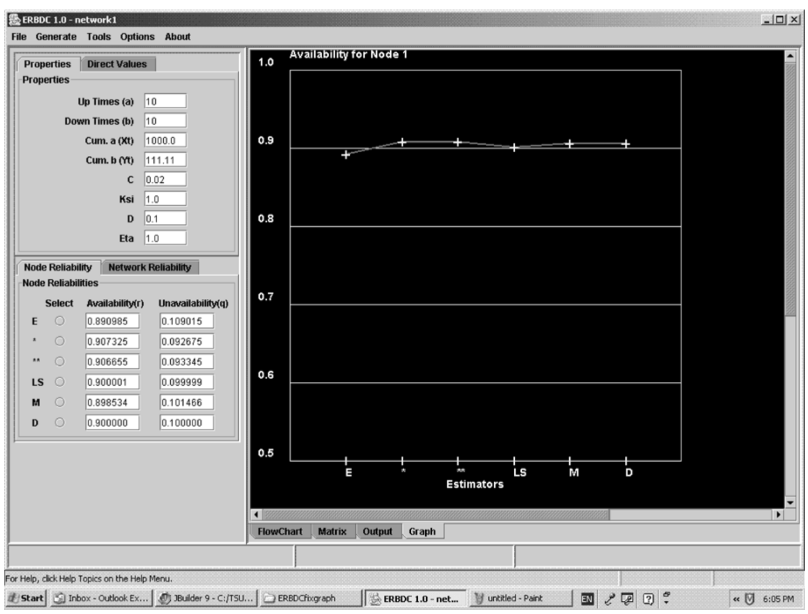

Fig.5. Comparison of availability estimators for a single component. (See Table I.)

are roughly exponentially distributed for most electronic hardware equipment. Let X_i and Y_j be the up- and down-times, respectively. Then, it follows:

$$f(x_i) = \lambda \exp(-\lambda x_i), \quad i = 1, 2, \ldots, a;\ \lambda > 0;\ x_i > 0 \quad \text{(A.1)}$$
$$f(y_i) = \mu \exp(-\mu y_i), \quad i = 1, 2, \ldots, b;\ \lambda > 0;\ y_j > 0 \quad \text{(A.2)}$$

where a = number of up times sampled and b = number of down times sampled. Now let the generator failure rate λ and the repair rate μ have independent prior distributions from the gamma family

$$\theta_1(\lambda) = \frac{\xi^c}{\Gamma(c)} \lambda^{c-1} \exp(-\lambda\xi); \quad \lambda > 0 \qquad \text{(A.3)}$$

where, for λ prior, c = shape parameter and ξ =inverse scale parameter, and

$$\theta_2(\mu) = \frac{\eta^d}{\Gamma(d)} \mu^{d-1} \exp(-\mu\eta); \quad \mu > 0 \qquad \text{(A.4)}$$

where, for μ prior, d = shape parameter and η = inverse scale parameter, are all estimated by means of a suitable prior estimation technique. The posterior distributions of λ and μ will be obtained by mixing their priors with the data. Since the family of gamma prior distributions for the failure rate λ and repair rate μ are conjugates to the exponential distributions of the "up" and "down" data, respectively, their respective posterior distributions will have the same gamma form with shape and scale parameters equal to the sum of the scale and shape parameters

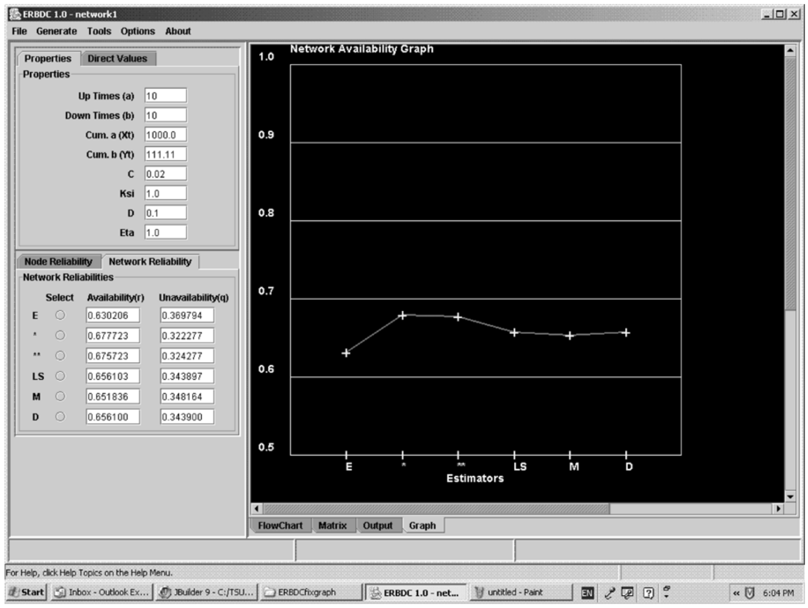

Fig.6. Comparison of availability estimators for a series network (four components).

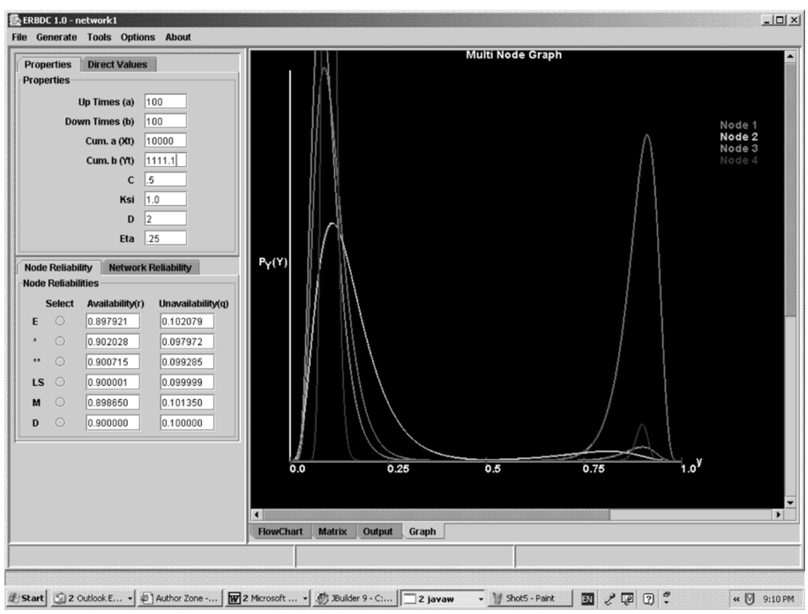

Fig.7. Density plots for the right skewed unavailability (on the left) and the left skewed availability (on the right) RVs.

of the prior and the current up- or down-time total. Therefore, from the sequence of (1)–(4), the joint likelihood of the uptime RVs is

$$f(x_1, x_2, \ldots, x_a|\lambda) = \lambda^a \exp(-x_T\lambda) \tag{A.5}$$

where $a =$ number of occurrences of "up" times sampled and $x_T =$ total sampled "up" time for a number of occurrences. The joint distribution of data and prior becomes

$$k(\underline{x}, \lambda) = f(x_1, x_2, \ldots, x_a; \lambda) = \frac{\xi^c}{\Gamma(c)}\lambda^{a+c-1}\exp\{-\lambda(x_T+\xi)\}. \tag{A.6}$$

Thus, the posterior distribution for λ is

$$\begin{aligned} h_1(\lambda|\underline{x}) &= \frac{k(\underline{x}, \lambda)}{\int_\lambda f(\underline{x}, \lambda)d\lambda} \\ &= \frac{\xi^c}{\Gamma(c)}\lambda^{a+c-1}\exp\{-\lambda(x_T+\xi)\} \\ &\quad \div \frac{\xi^c}{\Gamma(c)}(x_T+\xi)^{-1}\Gamma(a+c) \\ &= \frac{1}{\Gamma(a+c)}(x_T+\xi)\lambda^{a+c-1}\exp\{-\lambda(x_T+\xi)\} \end{aligned} \tag{A.7}$$

which is the Gamma $\{a+c, (x_T+\xi)^{-1}\}$ or gamma (n', $1/b'$) as earlier suggested due to conjugacy property. The same arguments hold for the repair rate μ. That is

$$h_2(\mu|\underline{y}) = \frac{1}{\Gamma(b+d)}(y_T+\eta)\mu^{a+c-1}\exp\{-\mu(y_T+\eta)\} \tag{A.8}$$

is the gamma $\{b+d,(y_T+\eta)^{-1}\}$ or gamma (m$'$, $1/a'$) posterior distribution for μ, where b = number of occurrences of "down" times sampled and y_T = total sampled "down" times for b number of occurrences, usually $a = b$.

Let Q be the random variable for FOR(Unavailability) $= q = (\lambda/(\lambda+\mu))$ Then derive its cdf, where

$$\begin{aligned} G_Q(q) &= P(Q \le q) \\ &= P\left(\frac{\lambda}{(\lambda+\mu)} \le q\right) \\ &= \text{AREA}_1 \text{ for } a \text{ given } 0 < q < 1. \end{aligned} \tag{A.9}$$

Now, use the property that gamma (n$'$, $1/b'$) has the moment generating function $(1-t/b')^{n'}$. This is the mgf of the chi-square distribution with 2n$'$ degrees of freedom. Then it follows:

$$\frac{\left(\frac{2a'}{2m'}\right)\mu \sim \frac{\chi^2_{2m'}}{2m'}}{\left(\frac{2b'}{2n'}\right)\lambda \sim \frac{\chi^2_{2n'}}{2n'}} = F_{2m',2n'}(F \text{ distribution with numerator } d.f. = 2m' \text{ and denominator } d.f. = 2n'. \tag{A.10}$$

It follows from (A.9) that by taking reciprocals of both sides and switching the inequality sign

$$\begin{aligned} G_Q(q) &= P\left(\frac{(\lambda+\mu)}{\lambda} \ge \frac{1}{q}\right) \\ &= P\left(1+\frac{\mu}{\lambda} \ge \frac{1}{q}\right) \\ &= P\left(\frac{\mu}{\lambda} \ge q^{-1}-1\right). \end{aligned} \tag{A11}$$

Multiplying both sides of (A.11) by $((2a'/2m')/(2b'/2n'))$, one obtains

$$\begin{aligned} G_Q(q) &= P\left\{\frac{\left(\frac{2a'}{2m'}\right)\mu}{\left(\frac{2b'}{2n'}\right)\lambda} > \frac{\frac{a'}{m'}}{\frac{b'}{n'}}(q^{-1}-1)\right\} \\ &= P\left\{F_{2m',2n'} > C_1 = \frac{a'n'}{b'm'}(q^{-1}-1)\right\} \\ &= \text{AREA}_2. \end{aligned} \tag{A.12}$$

In other words, we obtain an equivalent AREA$_2$ for the solution of $P(F_{2m',2n'} > C_1)$ in (A.12), instead of attempting to calculate AREA$_1$ for (A.9) whose distributional form is not known or recognized. That is, note that AREA$_1$ = AREA$_2$. Now that we have an accurate representation of the cdf of Q, namely, $G_Q(q)$, let us find its mathematical expression by equating AREA$_1$ to AREA$_2$

$$G_Q(q) = 1 - G_{F2m',2n'}\left\{\frac{a'n'}{b'm'}(q^{-1}-1)\right\}. \tag{A.13}$$

Note that Snedecor's F-density is given by [12, p. 23]

$$f(F) = \frac{\Gamma\left[\frac{(m+n)}{2}\right]}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)}\left(\frac{m}{n}\right)^{\frac{m}{2}}\frac{F^{\frac{m-2}{2}}}{\left[1+\frac{m}{n}F\right]^{\frac{(m+n)}{2}}}, \quad 0 < F < \infty \tag{A.14}$$

where $\mu = E(F) = (n/n-2)$, for $n > 2$, and $\sigma^2 = \text{Var}(F) = (2n^2(m+n-2)/m(n-2)^2(n-4))$ for $n > 4$, and $F > 0$.

Since (A.13) is differentiable, using (A.14) and differentiating with respect to q through obeying the "chain rule" leads to (note, $m' = m/2$ and $n' = n/2$)

$$\begin{aligned} g_Q(q) &= -g_{F2m',2n'}\left\{\frac{a'n'}{b'm'}(q^{-1}-1)\right\} * \left[-\frac{a'n'}{b'm'}\left(\frac{1}{q^2}\right)\right] \\ &= \frac{a'n'}{b'm'}\left(\frac{1}{q^2}\right) * \frac{\Gamma(m'+n')}{\Gamma(m')\Gamma(n')}\left(\frac{m'}{n'}\right)^{m'} \\ &\quad \times \frac{\left[\frac{a'n'}{b'm'}\left(\frac{1}{q}-1\right)\right]^{m'-1}}{\left[1+\frac{m'a'n'}{n'b'm'}\left(\frac{1}{q}-1\right)\right]^{m'+n'}}. \end{aligned} \tag{A.15}$$

Simplifying and rearranging through a number of intermediate steps

$$\begin{aligned} g_Q(q) &= \frac{\Gamma(m'+n')}{\Gamma(m')\Gamma(n')}\frac{a'^{m'}}{b'^{m'}}\frac{(1-q)^{m'-1}}{\left[1+\frac{a'}{b'}\left(\frac{1}{q}-1\right)\right]^{m'+n'}}\frac{1}{q^2q^{m'-1}} \\ &= \frac{\Gamma(m'+n')}{\Gamma(m')\Gamma(n')}\frac{(1-q)^{m'-1}}{qq^{m'}} \\ &\quad \times \left\{\frac{[(b'q+a'(1-q)]^{m'}[b'q+a'(1-q)]^{n'}}{\left[b'q\frac{a'}{b'}\right]^{m'}[b'q']^{n'}}\right\}^{-1} \end{aligned} \tag{A.16}$$

$$g_Q(q) = \frac{\Gamma(m'+n')}{\Gamma(m')\Gamma(n')}a'^{m'}b'^{n'}\frac{(1-q)^{m'-1}q^{n'-1}}{[a'+q'(b'-a')]^{m'+n'}}. \tag{A.17}$$

Resubstituting for $n' = a+c$, $m' = b+d$, $b' = \xi + x_T$, and $a' = \eta + y_T$

$$\begin{aligned} g_Q(q) &= \frac{\Gamma(a+b+c+d)}{\Gamma(a+c)\Gamma(b+d)}(\eta+y_T)^{b+d}(\xi+x_T)^{a+c} \\ &\quad \times \frac{(1-q)^{b+d-1}q^{a+c-1}}{[\eta+y_T+q(\xi+x_T-\eta-y_T)]^{a+b+c+d}} \end{aligned} \tag{A.18}$$

is the pdf of the RV $0 < Q < 1$, as defined above for the underlying distributional assumptions stated.

APPENDIX B

Given a weighted squared error loss function for an unknown parameter θ and estimator $t = T(\underline{x})$, where the sample data vector $\underline{x} = x_1, x_2, \ldots x_n > 0$, $\theta > 0$, and weight function is $w(\theta)$, as such

$$L(\theta, t) = w(\theta)(\theta - t)^2. \tag{B.1}$$

Assuming that the prior of θ is $\lambda(\theta)$, the joint density of prior and $f(x)$ is given by

$$f(\underline{x}|\theta)\lambda(\theta). \tag{B.2}$$

Then, the conditional (posterior) distribution of θ given $\underline{x}$ is as follows:

$$k(\theta|\underline{x}) = \frac{f(\underline{x}|\theta)\lambda(\theta)}{\int_{\Omega} f(\underline{x}|\theta)\lambda(\theta)d\theta} \tag{B.3}$$

$$E\left[L(\theta,t)\right] = \int_{\Omega} w(\theta)(\theta-t)^2 k(\theta|\underline{x})d\theta. \tag{B.4}$$

Bayes solution is the minimum of the Bayes risk $= E[L(\theta,t)]$, for which

$$\frac{dE\left[L(\theta,t)\right]}{dt} = -\int_{\Omega} w(\theta)2(\theta-t)k(\theta|\underline{x})d\theta = 0 \tag{B.5}$$

$$\int_{\Omega} \theta w(\theta)k(\theta|\underline{x})d\theta = \int_{\Omega} tw(\theta)k(\theta|\underline{x})d\theta \tag{B.6}$$

$$t = T(\underline{x}) = \frac{\int_{\Omega} \theta w(\theta)k(\theta|\underline{x})d\theta}{\int_{\Omega} w(\theta)k(\theta|\underline{x})d\theta} = \frac{E\left[\theta w(\theta)|\underline{X}=\underline{x}\right]}{E\left[w(\theta)|\underline{X}=\underline{x}\right]}. \tag{B.7}$$

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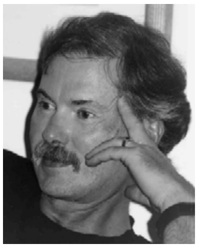

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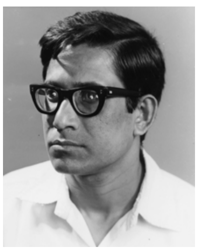

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