An Empirical Bayesian Stopping Rule in Testing and Verification of Behavioral Models

Mehmet Şahinoğlu, *Senior Member, IEEE*

***Abstract*—Software stopping rules are tools to effectively minimize the time and cost involved in software testing. The algorithms serve to guide the testing process such that if a certain level of branch or fault (or failure) coverage is obtained without the expectation of further significant coverage, then the testing strategy can be stopped or changed to accommodate further, more advanced testing strategies. By combining cost analysis with a variety of stopping-rule algorithms, a comparison can be made to determine an optimally cost-effective stopping point. A novel cost-effective stopping rule using empirical Bayesian principles for a nonhomogeneous Poisson counting process compounded with logarithmic-series distribution (LSD) is derived and satisfactorily applied to digital software testing and verification. It is assumed that the software failures or branches covered, whichever the case may be, clustered at the application of a given test-case are positively correlated, i.e., contagious, implying that the occurrence of one software failure (or coverage of a branch) positively influences the occurrence of the next. This phenomenon of clustering of software failures or branch coverage is often observed in software testing practice. The r.v. w_i of the failure-clump size of the interval is assumed to have LSD(θ) and justified on the data sets by employing a chi-square goodness of fit testing while the distribution of the number of test cases is Poisson(λ). Then, the distribution of the total number of observed failures, or similarly covered branches, X is a compound Poisson ^ LSD, i.e., negative binomial distribution, given that a certain mathematical identity holds. For each checkpoint in time, either the software satisfies a desired reliability attached to an economic criterion, or else the software testing is allowed to continue. By using a one-step-look-ahead formula derived for the model, the proposed stopping rule is applied to five test case-based data sets acquired by testing embedded chips through the complex VHDL models. Further, multistrategy testing is conducted to show its superiority to single-stage testing. Results are satisfactorily interpreted from a practitioner's viewpoint as an innovative alternative to the ubiquitous test-it-to-death approach, which is known to waste billions of test cases in a tedious process of finding more bugs. Moreover, the proposed dynamic stopping-rule algorithm can validly be employed as an alternative paradigm to the existing on-line statistical process control methods static in nature for the manufacturing industry, provided that underlying statistical assumptions hold. A detailed comparative literature survey of stopping-rule methods is also included in terms of pros and cons, and cost effectiveness.**

***Index Terms*—Bernoulli process, cluster effect, compound Poisson process, cost effective, effort domain, empirical Bayesian analysis, failure or branch coverage, logarithmic-series distribution (LSD), negative binomial distribution (NBD), positive autocorrelation, stopping rule.**

Manuscript received December 15, 2002; revised July 9, 2003.

The author is with the Department of Computer and Information Science, Troy State University Montgomery, Montgomery, AL 36103-4419 USA (e-mail: mesa@tsum.edu).

Digital Object Identifier 10.1109/TIM.2003.818548

I. Introduction and Motivation

THIS PAPER describes a statistical model to devise a stopping criterion for random testing in VHDL based hardware verification. The method is based on statistical estimation of branching coverage and will flag the stopping criteria to halt the verification process or to switch to a different verification strategy. The paper gives some results on some VHDL descriptions. This paper builds upon the statistical behavior of failure (or fault) or branch coverage described in Section II. Applying empirical Bayesian and other statistical methods to problems in hardware verification, such as better stopping rules, should be a fruitful area of research where improvements in the state of the art would be very valuable. Technically, the general concept is questionable. However, the stopping-rule idea is generally accepted to be more rational than having no value-engineering judgment to stop testing, as often dictated by a commercially tight time-to-market approach [41]. There is actually a large number of research and practical results available in statistically analyzing hardware verification processes. All major microprocessor companies heavily rely on such concepts. Note, faults and failures are taken to be synonymous here for convenience.

When designing a VLSI system in the behavioral level, one of the most important steps to be taken is verifying its functionality before it is released to the logic/PD design phase. It is widely believed that the quality of a behavioral model is correlated to the experienced branch or fault coverage during its verification process [17]–[19], [31], [51]. However, measuring coverage is just a small part of ensuring that a behavioral model meets the desired quality goal. A more important question is how to increase the coverage during verification to a certain level with a given time-to-market constraint. Current methods use brute force where billions of test cases were applied without knowing the effectiveness of the techniques used to generate these test cases [17]–[19], [32], [46]. One may consider behavioral models as oracles in industries to test against when the final chip is produced. In this work, in experimental sets involved, branch coverage (in five data sets of DR1 to DR5) is used as a measure for the quality of verifying and testing behavioral models. Minimum effort for achieving a given quality level can be realized by using the above proposed empirical Bayesian stopping rule. The stopping rule guides the process to switch to a different testing strategy using different types of patterns, i.e., random versus functional, or using different set of parameters to generate patterns or test cases or test vectors when the current strategy is expected not to increase the coverage. This leads to the practice of mixed-strategy testing. One can demonstrate the use of the stopping-rule algorithm on complex VHDL models. One has observed that switching phases at certain points guided by the stopping rule would yield to the same or even better coverage with less number of testing patterns. This method is an innovative alternative to help save millions of test-patterns and hence reduce cost in the colossal testing process of embedded chips versus the conventionally used "test-it-to-death" exhaustive-testing approach which wastes billions of vectors hoping to find more bugs in fault testing or cover more branches, all leading to a tedious process.

There occur many physical events according to the independent Poisson process, and, for each of these Poisson events, one or more other events can occur. This is identified as over-dispersion in many life-sciences oriented textbooks, to cover the total number of certain bacteria or algae clustered on individual leaves in a water pond to exemplify [3], [4]. If an interruption during the testing of a software program is assumed to be due to one or more software failures (or branch coverage) in a clump, and if also the distribution of the total number of interruptions or test cases is Poisson; then the distribution of total number of experienced failures, or covered branches is a compound Poisson (CP) [6], [8]–[15]. The empirical Bayesian stopping rule, therefore, uses the mathematical principles of a Poisson counting process as applied to the count of test cases, with a logarithmic-series distribution (LSD) applied to the cluster size of software failures or branch coverage generated by each test-case. It satisfactorily applies to a time-continuous, compounded and nonhomogenous Poisson process, as well as to a time-independent effort (or test-case) based testing such as in a sequentially discrete Bernoulli process. Namely, Poisson process is a time-parameter version of the counting process for Bernoulli trials ([20], p. 72). It is imperative to recall that often used Binomial processes are the sum of identical Bernoulli distributed random variables. However, those Bernoulli random variables at each test-case epoch are nonidentical with unequal "arrival" success probabilities as earlier studied by Sahinoglu in a 1990 publication [9]. The proposed model assumes randomization of test cases in the spirit of independently incremented Poisson counting process, since the coverage sizes do not necessarily follow a definite trend unless test cases are ordered in a merit order. This is a practice, which is impossible to attain perfectly prior to actual experimentation. Some sources claim that the independent-increments Poisson arrival model is applicable for the first "surprise" execution against a test suite. On second and subsequent executions, the "arrival" (or discovery) of faults (or branches) is no longer random unless the software development process is chaotic or parallel-distributed. Evidently, the applicability of such an independent-increments counting process and hence the proposed stopping rule varies with the maturity of the software testing activity being developed. This is why the regression testing techniques to observe for the said maturity is of relevance here in terms of mainstream software engineering [42]. Also, some authors support the concept of probability distribution function, $\mathrm{p(t)}$: interruption correlation function, for the occurrence of interruptions, a concept which is rather hazy and nebular [33]. First, the total number of observations should always be known in advance to model the probability of interruptions, which testers are unable to master. Therefore, $\mathrm{p(t)}$ is an unrealistic guesswork and it also clearly varies from one data to another, strictly not to be generalized. It is therefore more rational to randomize statistically the interruption activity, which is so much more natural as unprecedented test cases may act surprisingly different at random epochs. The randomization phenomenon is also in the spirit of a Poisson process with independent increments on which MESAT tool is structured. The unpredictability factor of fault-arrival or branch-coverage is therefore best addressed by a nonhomogenous Poisson process whose rate of arrival is adjusted, in this case diminished, with the advance of time or number of test cases. This nonstationarity of a Poisson process takes care of the no-longer independent Poisson arrival times, a phenomenon best displayed by the NHPP ([20], pp. 94–101).

When a new computer software package is written, compiled and all obvious software failures are removed for customary input sets; then, a testing program is usually initiated to eliminate the remaining failures. The common procedure is to use the software package on a set of problems, and whenever the testing is interrupted because of one or more programming failures, the codes are corrected, the software re-compiled, and computation is re-started. This type of testing can continue for several time units (hours, days, or weeks, etc.) with the number of failures per unit time lessening. The same is true for instance when discretely applied test cases replace test weeks and branch coverage records replace those of failure coverage. Finally, one reaches a point of optimal economic return in time or effort when testing is stopped and the software released. However, one is never certain that all software faults due to failures have been removed, or similarly all branches have been covered. Although there may still be a small number of failures remaining in the software, the chances of finding them within a reasonable time may be so small that it is not economically feasible to continue testing [6], [21]. The objective is to find a cost-effective stopping rule to terminate testing. One can add the dimension of a preconceived confidence-level, $0 < \mathrm{CL} < 1$, to ensure minimal coverage reliability.

Stopping-rule problem has been studied extensively by statisticians and engineers [2], [7], [34], [37]–[40], [44]. In this research paper, however, a cost-effective stopping rule is presented with respect to a popularly used one-step-look-ahead economic criterion when an alternative underlying pdf is assumed for the clump size for the failures or branches observed. The total number of discovered failures or covered branches is the Poisson counting process compounded with the LSD at each Poisson arrival. That is, the number of incidents over time is distributed as Poisson, whereas the number of failures that occur as a clump at each interruption epoch or incident is distributed according to a discrete LSD. The failures within a clump are positively correlated with each other. This phenomenon is represented by a parameter $0 < \theta < 1$ in the LSD for the clump size r.v. A Poisson distribution compounded by a discrete LSD will be denoted as a Poisson ^ LSD, namely a negative binomial distribution (NBD) pending a certain mathematical identity as in (14) below. The algorithm is applied in effort domain where test cases are applied for the five data sets experimented on embedded chips [17]–[19], [41], [44].

II. Notations, Compound Poisson, and Empirical Bayes Estimation

r.v.	Random variable.
pdf	Probability density function.
iid	Independent and identically distributed.
CP	Compound Poisson distribution.
CPSRM	Compound Poisson software reliability model.
JPL	Jet Propulsion Laboratory.
LSD	Logarithmic-series distribution.
EDA	Exploratory data analysis.
MESAT	Proposed stopping-rule java applet.
CL	Confidence level, or a minimal percentage of branches or failures to cover.
Poisson ^ LSD	Poisson distribution compounded by LSD.
NBD	Negative binomial distribution.
$\mathrm{N(t)}$	The r.v. for the number of Poisson events until and including time "t".
$\mathrm{X(t)}$	Total number of failures distributed w.r.t. Poisson ^ LSD until discrete time unit t.
$\mathrm{w_i}$	The r.v. of failure clump size, distributed w.r.t. LSD at each Poisson event i.
$\Theta = \theta$	LSD parameter which denotes the positive correlation. θ is a realization of the r.v. Θ.
a	Constant for the LSD r.v. of w.
k	NBD parameter (calculated recursively at each Poisson epoch).
λ	Poisson rate or parameter where $\lambda = -k\ln(1-\theta) = k\ln q$ holds.
θ_1	Lower limit of θ.
θ_2	Upper limit of θ.
cf. or $\Phi_{X(t)}$	Characteristic function of $\mathbf{X}(\mathrm{t})$.
$\mathrm{dif}(\theta)$	Range for LSD parameter, the correlation coefficient: $\theta_2 - \theta_1$.
q	Reciprocal of $(1-\theta)$. When $\theta = 0$, no compounding phenomenon exists; the process is then purely Poisson with $\mathrm{q} = 1$. As $\mathrm{q} > 1$ increases, the compounding or over-dispersion effect also increases.
p	Related parameter, $\mathrm{p} = \mathrm{q} - 1$. No compounding or pure Poisson when $\mathrm{p} = 0$.
$\mathrm{f(X \mid \theta)}$	Discrete negative-binomial conditional probability distribution of X.
$\mathrm{h}(\theta)$	Prior distribution of the positive correlation parameter.
$\mathrm{h(X)}$	Marginal distribution of X following the Bayesian analysis.
$\mathrm{Beta}(\alpha, \beta)$	Prior Beta distribution for LSD variable.
α, β	Positive shape and scale parameters of Beta.
$\mathrm{h}(\theta \mid \mathrm{X})$	Posterior conditional distribution of θ after updating for X: failure vector.
$\mathrm{f}(\mathrm{X} \mid \theta)$	Discrete conditional probability distribution of X given Θ.
$\mathrm{E}(\theta \mid \mathrm{X})$	Bayes estimator w.r.t. squared error loss function. Expected value of the conditional posterior r.v. θ.
$\mathrm{E(X) = kp}$	Expected value of the conditional $\mathrm{X} \sim$ NBD whose only parameter is k and based on a single r.v., $\Theta \sim \mathrm{h}(\theta \mid \mathrm{X})$ which is conditional posterior.
$\mathrm{S}(\cdot) = \mathrm{s}$	The stopping-rule S gives the number of failures "s" to stop after $(\cdot)$ many discrete time units (days, weeks, etc.) or number of test cases.
C	Combination (n, k) notation to denote how many different unordered combinations of "size k out of a sample of n" exist.
DR1-5	Effort-based time-independent (using test cases) coverage data sets 1 to 5.

A nonstationary CP arrival process is given in [6], [9]–[14], and ([20], pp. 90—101)

$$\{\mathrm{X(t)}, \mathrm{t} \geq 0\} = \sum_{i=1}^{N(t)} w_i \tag{1}$$

where, $\mathrm{N(t)} > 1$ and the compounding clump sizes $\mathrm{w_1, w_2}, \ldots$ are iid and, where $\mathrm{f(w_i)}$ are distributed with LSD [8] (LSD) as follows:

$$\mathrm{f(w)} = \mathrm{a}\frac{\theta^w}{w}, \quad 0 < \theta < 1, \quad \mathrm{a} > 0, \quad \mathrm{w} = 1, 2, \ldots \tag{2}$$

and

$$\mathrm{a} = -\frac{1}{\ln(1-\theta)}. \tag{3}$$

Then, $\mathrm{X(t)}; \mathrm{t} \geq 0$ is a Poisson ^ LSD process when $\mathrm{N(t)} \sim \mathrm{Poisson}(\lambda)$ and $\mathrm{w_i} \sim \mathrm{LSD}(\theta)$ for $\mathrm{i} = 1, 2, \ldots [3, 4, 11, 13]$. However, if we let

$$\lambda = -k\ln(1-\theta) = k\ln q, \quad \mathrm{k} > 0 \tag{4}$$

where

$$q = \frac{1}{1-\theta} \tag{5}$$

then, $\mathrm{X} \sim$ Poisson ^ LSD, is an r.v. with NBD. $\mathrm{E(X) = kp}$ when $\mathrm{E(X)}$ is the number of expected failures within the next time or effort unit. Since

$$\begin{aligned}\mathrm{f(w)} &= \frac{1}{\ln q} \cdot \frac{1}{w} \left(\frac{q-1}{q}\right)^{\mathrm{w}} \\ &= \frac{1}{\ln q} \cdot \frac{1}{\mathrm{w}} \left(\frac{p}{q}\right)^{\mathrm{w}}\end{aligned} \tag{6}$$

where

$$\mathrm{p} = \mathrm{q} - 1, \quad \mathrm{q} = (\mathrm{p} + 1) > 1 \tag{7}$$

and its characteristic function (cf.) is derived as follows:

$$\Phi_{X(t)}(\mathrm{u}) = \exp\{\lambda(\phi_w(u) - 1)\} \tag{8}$$

where $\phi_w(u)$ is the cf. of LSD which is given by

$$\phi_w(u) = 1 - \frac{1}{\ln q}\{\ln(\mathrm{q} - \mathrm{pe^{iu}})\}. \tag{9}$$

Then

$$\Phi_{X(t)}(u) = \exp\left\{\mathrm{k}\ln\mathrm{q}\left(1 - \frac{1}{\ln q}(\mathrm{q} - \mathrm{pe^{iu}}) - 1\right)\right\}$$
$$= \exp\{\ln(\mathrm{q} - \mathrm{pe^{iu}})^{-\mathrm{k}}\}$$
$$= (\mathrm{q} - \mathrm{pe^{iu}})^{-\mathrm{k}}. \quad (10)$$

Note, $\Phi_{X(t)}(u)$ is the cf. of NBD. Now, the probability distribution function of X is

$$\mathrm{f(X)} = \mathrm{C_{k-1}^{k+X+1}}\frac{p^x}{q^{k+X}} \quad (11)$$

where C denotes combination operator and from (7)

$$\mathrm{p} = \mathrm{q} - 1 = \frac{1}{1-\theta} - 1 = \frac{\theta}{1-\theta} \quad (12)$$

where $\mathrm{q} = (1/1-\theta)$. Thus, reorganizing (11)

$$\mathrm{f(X}\,|\,\theta) = \mathrm{C_{k-1}^{k+X-1}}\left(\frac{\theta}{1-\theta}\right)^X (1-\theta)^{\mathrm{k+X}}. \quad (13)$$

Since the positive autocorrelation among the failures or branches in a cluster is not constant, and for it varies from one to another, it can be well treated as a random variable denoted by θ that ranges from 0 to 1. Hence, among continuous distributions with a range between 0 and 1, the Beta distribution can be considered as a conjugate prior distribution for θ with its corresponding pdf. Since $0 < \theta < 1$, we let the prior pdf of the r.v. $\Theta = \theta$ to be a Beta (α, β) pdf with

$$\mathrm{h}(\theta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1},$$
$$0 < \theta < 1; \quad \alpha, \beta > 0 \quad (14)$$
$$\mathrm{f(X}\,|\,\theta) = \mathrm{C_{k-1}^{k+X-1}}\theta^X(1-\theta)^k. \quad (15)$$

Then, the joint pdf of $\mathrm{X} = \sum_{i=1}^{N(t)} w_i$ and Θ is given as follows:

$$\mathrm{h}(\theta, \mathrm{X}) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\theta^{X-1}(1-\theta)^{\beta-1}\mathrm{C_{k-1}^{k+X-1}}\theta^X(1-\theta)^k \quad (16)$$
$$= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\mathrm{C_{k-1}^{k+X-1}}\theta^{\alpha+X-1}(1-\theta)^{\beta+k-1}$$

and whereas the marginal distribution of X is given as

$$\mathrm{h(X)} = \mathrm{C_{k-1}^{k+X-1}}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\int_0^1 \theta^{\alpha+X-1}(1-\theta)^{\beta+k-1}\,\mathrm{d}\theta$$
$$= \mathrm{C_{k-1}^{k+X-1}}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\frac{\Gamma(\alpha+\beta+X+k)}{\Gamma(\alpha+X)\Gamma(\beta+k)}. \quad (17)$$

Now, by using the Bayes' theorem [2], [16] where $\mathrm{h}(\theta\,|\,\mathrm{X}) = [\mathrm{f(X}\,|\,\theta)\cdot\mathrm{h}(\theta)]/\mathrm{h(X)}$, the posterior distribution of $(\theta\,|\,\mathrm{X})$ is derived as follows:

$$\mathrm{h}(\theta\,|\,\mathrm{X}) = \frac{\Gamma(\alpha+X)\Gamma(\beta+k)}{\Gamma(\alpha+\beta+X+k)}\theta^{\alpha+X-1}(1-\theta)^{\beta+k-1} \quad (18)$$

and this is a well-known Beta distribution, as in (19)

$$\mathrm{h}(\theta\,|\,\mathrm{X}) = \mathrm{Beta}(\alpha+X, \beta+k). \quad (19)$$

With respect to squared-error loss function definition [16], its expected value is given as

$$\mathrm{E}(\theta\,|\,\mathrm{X}) = \frac{\alpha+X}{\alpha+\beta+X+k} \quad (20)$$

which is the defined to be the Bayes estimator. We know that the expected value of the r.v. X, which is an NBD, is given as in the following by substituting the Bayes posterior pdf of θ from (19) into (21), then using (12) for p, and $\mathrm{E(X)} = \mathrm{kp}$

$$\mathrm{E(X)} = \mathrm{k}\int_0^1 \frac{\theta}{1-\theta}\mathrm{h}(\theta\,|\,\mathrm{X})\,\mathrm{d}\theta \quad (21)$$
$$\mathrm{E(X)} = \mathrm{k}\left(\frac{\alpha+X}{\beta-1+k}\right). \quad (22)$$

Therefore $\lambda = -k\ln(1-\theta) = k\ln q$ and thus $\mathrm{k} = (\lambda/\ln q)$ from (4) can be approximated recursively as in (23), when the posterior Bayes estimator of θ from (20), i.e., $\mathrm{E}(\theta\,|\,\mathrm{X}) = (\alpha + x)/(\alpha+\beta+X+k)$ is entered for θ in (4) and

$$\mathrm{e}^{\frac{\lambda}{k}} = \frac{\alpha+\beta+X+k}{\beta+k} \quad (23)$$

which is a nonlinear equation that can be readily solved using Newton–Raphson method, employing an initial $\mathrm{k}(0)$. Since α and β are given constants, and at each discrete step, we use the accumulated X (total failure or branch coverage) and calculate the constant "k" for the next step.

However, using the generalized (incomplete) Beta prior [5] instead of the standard Beta prior can be more reasonable and realistic since the former includes the expert opinion (sometimes called an "educated guess") about the feasible range of the parameter $0 < \theta < 1$. Therefore, θ can be entered by the analyst as a range or difference this time in the form of $\mathrm{dif}(\theta) = \theta_2\ (\mathrm{upper}) - \theta_1\ (\mathrm{lower})$ to reflect a range of a prior belief of positive correlation among the software failures or branches covered in a clump. And finally, we derive a more general (24) for the "generalized beta" to replace the earlier (23) that was derived for the "standard beta prior." The author can be contacted for a detailed derivation of (24) and consequently (27)

$$\mathrm{e}^{\frac{\lambda}{k}} = \frac{(\alpha+\beta+X+k)}{(1-\theta_2+\theta_1)(\alpha+X)+\beta+k}. \quad (24)$$

Therefore, (23) transforms into (24) for the generalized Beta.

For example, when $\theta_1 = 0$ and $\theta_2 = 0.60$, we will have $e^{(\lambda/k)} = (\alpha+\beta+X+k)/(0.40(\alpha+X+\beta+k))$.

One should emphasize that X is an input data to denote the experienced value of number of discovered failures or covered branches as a realization of the CP. Consequently, $\mathrm{E(X)}$ is the expected value of software failures or branch coverage in the next unit of time or discrete effort (test case). If $\mathrm{E(X)}$ is multiplied by the time units or efforts (test cases) remaining, than one predicts the expected number of remaining failures or branches.

III. Proposed Stopping Rule in Software Testing

If the expected incremental difference between sequential steps, $\mathrm{i} = 1, 2, \ldots$ where i denotes testing interval in terms of days, weeks in time-domain or test cases in effort domain, is illustrated to exceed a given economic criterion "d", then continue testing. Otherwise stop testing Observe below in one-step-look-ahead formula, whose utility is maximized (or loss minimized) as shown earlier in the publication by [6], [13], [21].

$$\mathrm{e(X)} = \mathrm{E(X_{i+1})} - \mathrm{E(X_i)} \le d \tag{25}$$

where, (25) can be rearranged in the form of (26) by utilizing (24)

$$\mathrm{e(X)} = \mathrm{k_{i+1}} \frac{(\alpha + X_{i+1})}{(\beta - 1 + k_{i+1})} - \mathrm{k_i} \frac{(\alpha + X_i)}{(\beta - 1 + k_i)} \le \mathrm{d}. \tag{26}$$

However, incorporating the Generalized Beta, as shown by (27) at the bottom of the page, where $\mathrm{d} = (c/a - b)$ and $\alpha, \beta, \mathrm{k_i}, \mathrm{X_i}, \theta_2$, and θ_1 are input values at each discrete step i.

Note that (27) defaults to (26) for $\theta_1 = 0$ and $\theta_2 = 1$ or $\mathrm{diff}(\theta) = \theta_2\ (\mathrm{upper}) - \theta_1\ (\mathrm{lower}) = 1$, when neither an expert judgment or nor an educated guess exists on the bounds of correlation strength for failure clumps. If we were to stop at a discrete interval i, we assume that the discovered failures or branch coverage will have to accrue in the field a cost of "a" per failure or branch after the fact or following the release of the software. Thus, there is an expected cost over the interval $\{\mathrm{i}, \mathrm{i}+1\}$ of $\mathrm{aE\{X_i\}}$ for stopping at time $\mathrm{t} = \mathrm{i}$ or test-case i. If we continue testing over the interval, we assume that there is a fixed cost of "c" for testing, and a variable cost of "b" of fixing each failure found during the testing before the fact or preceding the release of the software. Note that "a" is usually larger than "b" since it should be considerably more expensive to fix a failure (or recover an undiscovered branch) in the field than to observe and fix it while testing. Thus, the expected cost for the continuation of testing for the next time interval or test-case is $\mathrm{bE(X_i)} + \mathrm{c}$. This cost model [6] is somewhat similar, if not exactly the same, to that of criterion expressed in [1]. Opportunity or shadow cost is not considered here since such an additional or implied cost may be included within a more expensive and remedial after-release cost coefficient denoted by "a." Some researchers are not content with these fixed costs. However, the MESAT tool employed here can treat that problem through a "variable costing" data driven approach as needed by the testing analyst. That is, a separate value is entered in the MESAT java applet for a or b or c at each test case at will, if these cost parameters are defined to vary from test-case to test-case.

Therefore, an alternative cost model similar to that of Dallal and Mallows is used [1], [6]. If for the ith unit interval beginning at time t or for the ith test-case, the expected cost of stopping is greater than or equal to the expected cost of continuing, i.e., therefore

$$\mathrm{aE(X_{i+1})} \ge \mathrm{bE(X_i)} + \mathrm{c} \tag{28}$$

then, it is economical to continue testing through the interval or effort. On the other hand, if the expected cost of stopping is less than the expected cost of continuing (when inequality sign is reversed), it is more economical and cost effective to stop testing

$$\mathrm{aE(X_{i+1})} < \mathrm{bE(X_i)} + \mathrm{c}. \tag{29}$$

The decision theoretic justification for this stopping rule is trivially simple. When $\mathrm{E(X_{i+1})} = \mathrm{E(X_i)}$ is almost identical at the point of equality or equilibrium where the decision of stopping has the most utility (lowest loss) due to negligible difference between the old and fresh information, we stop at a balance point between undertesting and overtesting. Then, (29) follows as in (30)

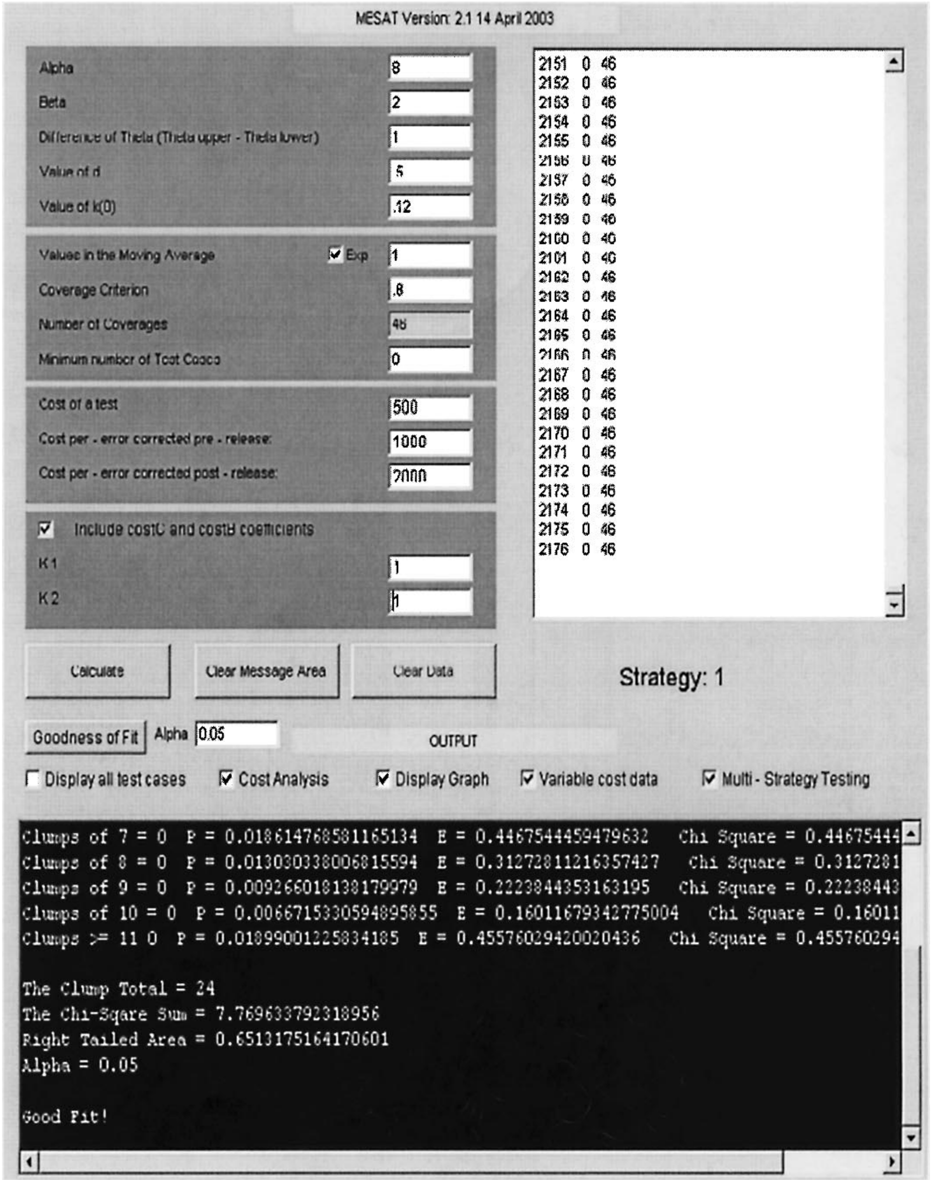
$$\mathrm{E(X_{i+1})} - \mathrm{E(X_i)} = \frac{c}{a-b} = \mathrm{d}. \tag{30}$$

However, this research paper also maintains that one-step-look-ahead decision is not the only way. A multistrategy such as a two-stage decision making is shown to be superior as also recently studied by Sahinoglu *et al.* [41]. This is equivalent to using the same stopping rule for the latent data following the decision made for the earlier stopping rules as McDaid and Wilson studied [38] based on Singpurwalla's and Wilson's taxonomy in their most recent book ([45], chapter 6). The (27) here is neither a fixed-time look-ahead nor a one-bug look-ahead plan as outlined in the same book [45]. However, it is a one-stage look ahead testing, further fortified by a second or third stage testing if needed, which is called a multistrategy testing plan, in this paper as supported in some recent publications by the author and his co-authors [17]–[19], [41], [44]. Screen 1 and Graphs 1 and 2 show the practical application of these multistrategy rules using the proposed look-ahead (27) under the newly proposed NBD probability model, which is a compounded NHPP.

The above stopping rule outline through (25)–(30) essentially state that, if the expected number of failures (or branch coverage) that can be found in the software in the next unit time or effort is sufficiently small, one should stop testing and release the software package to the end user. If the expected number of failures (branch coverage) is large, one should continue testing to cover more grounds. The stopping rule depends on an up-to-date expression for a Poisson ^ LSD distribution, or NBD given a special assumption holding. Therefore we need accurate estimates of θ to update stepwise. However, such estimates depend on the history of testing, which implies the use of an empirical Bayes decision procedures as described above, such as in the "statistician's reward" or "secretary" problem of the optimal stopping chapter where a fixed cost "c" per observation is considered [1], [2], [7], [37]–[40], [42].

Moreover, the divergence factor, $\mathrm{d} = (c/(a-b))$ in (30) signifies the ratio of the *cost "c"* of performing a test over the difference between the higher "a" cost of catching a failure *after the fact and* the lower "b" cost of catching a failure *before* release. Given, the numerator "c" is constant, intuitively, a

$$\mathrm{e}(\mathbf{X}) = \mathrm{k_{i+1}} \frac{(\theta_2 - \theta_1)(\alpha + X_{i+1})}{(\alpha + \beta - 1 + X_{i+1} + k_{i+1}) - (\theta_2 - \theta_1)(\alpha + X_{i+1})} - \mathrm{k_i} \frac{(\theta_2 - \theta_1)(\alpha + X_i)}{(\alpha + \beta - 1 + X_i + k_i) - (\theta_2 - \theta_1)(\alpha + X_i)} \le \mathrm{d} \tag{27}$$



Screen 1. MESAT Version 2.1.

large difference between "a" and "b," hence a smaller "d", will delay the stopping moment as it is costlier to stop prematurely by leaving uncorrected failures or undetected branches. Also, given the denominator "a-b" is constant, a smaller testing cost per test-case "c" yielding a smaller "d", will likewise delay the stopping moment as it is cheaper to experiment more. Moreover, $\alpha, \beta, \mathrm{k_i}, \mathrm{X_i}, \theta_2$, and θ_1 are input constants at each discrete step i, where, α and β are apriori parameters for the LSD(θ) in the Bayesian analysis, where $0 < \theta < 1$ denotes the positive-correlation-coefficient-like parameter θ of LSD. In (4) and (20), k is an unknown quantity. Note that θ and k together define the Poisson λ, which is an important parameter of the model. A complete Bayesian analysis requires an inference on k as well. Note that even though such analysis does not yield analytically tractable results, it can easily be done using Markov Chain Monte Carlo (MCMC) methods. Since k is not described probabilistically, but estimated using data, the approach followed is not fully a Bayesian, however an empirical Bayesian [43]. Also, MCMC is beyond the scope of this research paper that does not use a fully Bayesian approach.

θ_2 and θ_1 are upper and lower constraints for θ, if default situation $\theta_2 - \theta_1 = 1.0$ is not selected. Now, let RF = Remaining number of faults or coverage after the stopping action and RT = Remaining number of test cases after the stopping action. Then in order for the stopping-rule algorithm to be cost-efficient, the below equation all in \$ units should hold

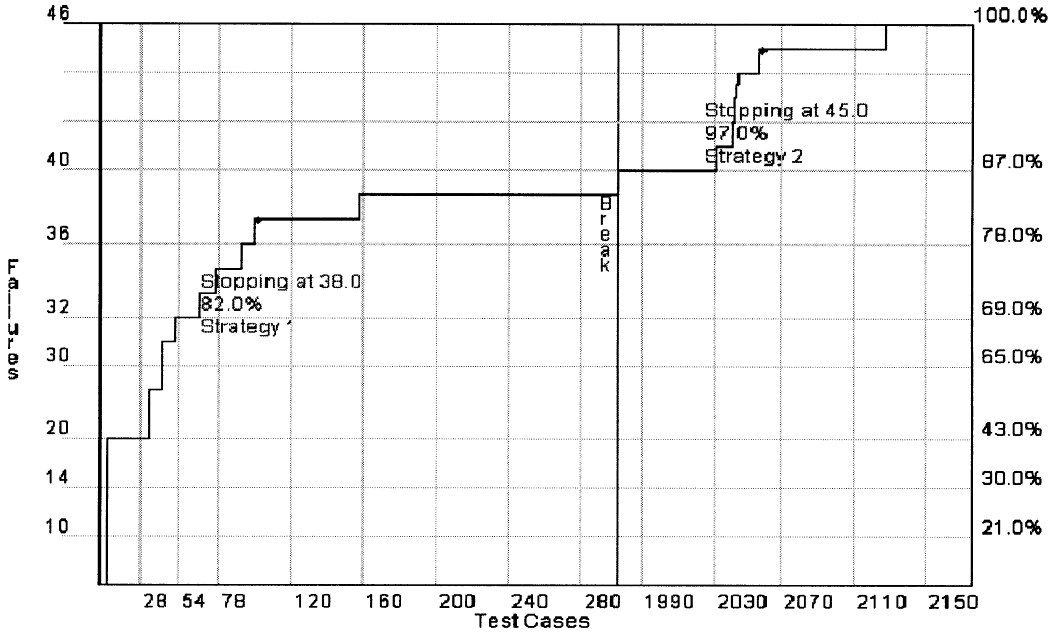
$$\mathrm{(RF)a} \leq \mathrm{(RF)b} + \mathrm{(RT)c} \tag{31}$$

from which, the inequalities for "a≤", "b≥," and "c≥" can validly be derived using simple algebra

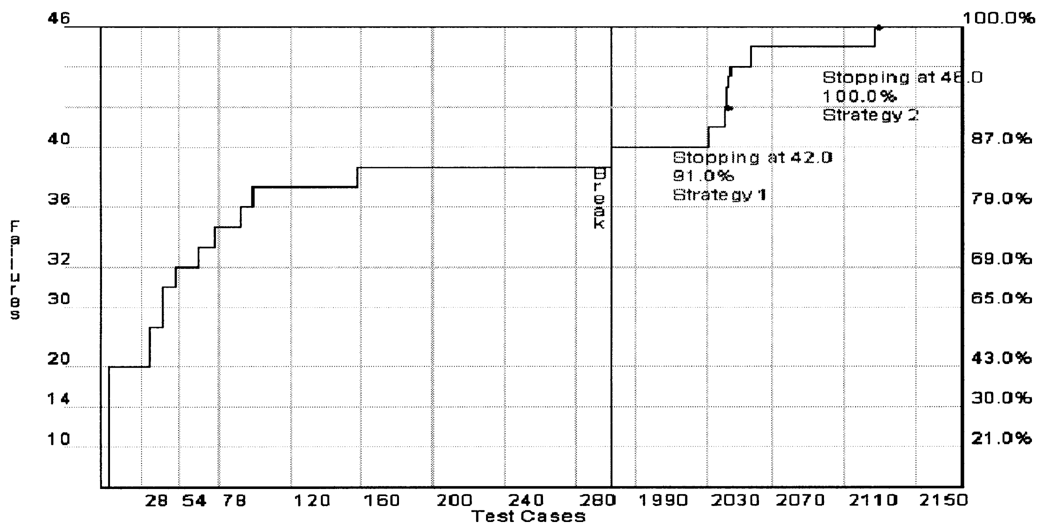
$$\mathrm{a} \leq \frac{\mathrm{(RF)}b + \mathrm{(RT)}c}{\mathrm{RF}} \tag{32}$$

$$\mathrm{b} \geq \frac{\mathrm{(RF)}a - \mathrm{(RT)}c}{\mathrm{RF}} \tag{33}$$

$$\mathrm{c} \geq \frac{\mathrm{(RF)}a - \mathrm{(RF)}b}{\mathrm{RT}}. \tag{34}$$



Graph 1. Plot of multistrategy stopping rule for DR5 at a minimum 80% confidence level.



Graph 2. Plot of multistrategy stopping rule for DR5 at a minimum 90% confidence level.

IV. Applications and Results

RHS of (31) is the dollar amount of savings due to stopping-action taken, by not executing the remaining test cases and by not correcting or detecting remaining faults (or branches). LHS of (31) is the dollar amount of potential loss if those remaining faults or coverage were to be corrected after release. If RHS is more than LHS in (31), then, it is a positive gain; otherwise, a negative loss. Let #TC = total number of test cases, #NC = total number of coverage, and #MC = minimum coverage required, which is equal to CL times #NC. Below in Table I are the 6 varying cost-scenarios for Table II that also indicates the subtle effect due to additional information on the range of θ.

There are five quadruplets in Table II, each signifying one data set, where each row in a quadruplet pertains to one of the four sensitivity studies for six cost-scenarios. Note that the first rows in each quadruplet demonstrate a test environment where the #TC is not available with therefore no confidence level (CL) specified. Thus, the testing halts whenever the one-step-look-ahead formula in (27) holds after at least two test cases with nonzero failures or branch coverage. Second rows in each quadruplet again possess no specified CL, but testing is allowed to continue until or past a certain given minimal number of test cases specified by the analyst and denoted by #TC due to an availability of budget resources, when again (27) is first verified.

To exemplify in Table II, for DR5's first row, stop at second test-case after covering four branches when (27) is first verified. For DR5's second row, when CL did not apply due to final number of failures or branches unknown, at least a typical prescribed minimal 1094 test cases were allowed to run at which the decisive (27) was also verified. Third and fourth rows in each quadruplet behave with respect to a confidence level of 0.80 (or 80%) and 0.9 (or 90%), respectively. Testing may halt on or after ensuring this specified minimal confidence level of coverage, since the total number of failures or branches is available. #TC in rows 3 and 4 simply display the total prescribed number of test cases for each data set. For DR5s 3rd

TABLE I
SIX COST SCENARIOS AND THEIR SENSITIVITY STUDIES FOR TABLE II

(1)a≤: Given c=$100, b=$1000; what is the optimal a≤ to render MESAT cost-efficient?
(2)a≤: Given c=$2000, b=$1000; what is the optimal a≤ to render MESAT cost-efficient?
(3 b≥: Given c=$10, a=$20000; what is the optimal b≥ to render MESAT cost-efficient?
(4 c≥: Given a=$5000, b=$1000; what is the optimal c≥ to render MESAT cost-efficient?
(5)Saved: Given c=$500, b=$1000, a=$2000, d=.5; what is the dollar amount saved ?
(6)Lost: Given c =$100, b=$10000 a=$20000, d=.1; what is the dollar amount lost ?

row, testing stops at 100th test-case for a $\mathrm{CL} = 0.8$ after covering 38 branches, exceeding the $\#\mathrm{MC} = 37$, which is found by $(\mathrm{CL}) * (\#\mathrm{NC}) = 0.8 * 38 = 36.4$. Also, "a" per undiscovered fault should be at most $26 950 according to the scenario (1), "b" should be at least $17 405 for the scenario (3), and "c" should be at least $15, in order for the stopping rule to be cost effective. Total savings is $+\$1\ 030\ 000$ due to scenario "(5)Saved" with the assumed cost parameters as shown in Table II and Screen 1. For DR5s 4th row, with $\mathrm{CL} = 0.9$, one stops at 2042nd test-case covering 42 branches to save $+\$63\ 000$.

The body of test cases is essentially randomized as in the major assumption of Poisson or Bernoulli counting processes. Savings $(+)$ or losses $(-)$ are definitely a function of the cost parameters involved in each scenario. Essentially, if the cost of redeeming coverage (failure or branch) is high, then it is disadvantageous to stop prematurely with respect to a stopping-rule algorithm, such as in MESAT. If the cost parameters are not known, then a sensitivity analysis can be conducted to observe a range of losses or savings. MESAT enjoys the benefit of a confidence level (CL) at will due to budget resources' availability, in addition to a one-step-ahead-criterion (27) controlled by a divergence factor, "d." Moreover, the MESAT algorithm effectively accounts for the clumping of the coverage, as well as the positive autocorrelation among the observations in an aggregate. MESAT is also flexible when the final number of coverage may not be known as exemplified in Table II, where you allow a minimal number of test cases to run. This method is also flexible for employing variable cost values, "a", "b, " or "c",, at different test cases across the spectrum, where some test cases may have more weight than others.

Note that in Table II, $\mathrm{dif}(\theta) = 1$ implies the usage of default standard Beta prior, whereas $\mathrm{dif}(\theta) \neq 1$ implies the implementation of the generalized Beta prior treated. It is clear that as the economic stopping criterion "d" varies from a liberal (higher) to a conservative (lower) threshold, the stopping rule is shifted and postponed to a later test-case, if not the same. By a conservative set-up, we mean a scenario where the stopping rule is trying not to miss any failures and testing activity is likely to stop later, rather than sooner. The correlation behavior within each clump is represented by our choice of α and β in the light of previous engineering judgment. Note that for $\alpha > \beta$, like in $\alpha = 8$ and $\beta = 2$ as imposed in the empirical Bayesian sense in the examples of Table II, the posterior r.v. of θ displays a distinctly left-skewed behavior. It has been observed that the stopping occurs earlier in this scenario. However, in $\alpha = \beta$ as in e.g., $\alpha = 5, \beta = 5$, where the Beta distribution looks evenly symmetrical as opposed to the presently skewed ones since $\alpha > \beta$, the correlation among coverage in each test-case is not that strong. In the latter case, it has been observed that the stopping rule then was delayed somewhat if not considerably. Therefore a choice of $\alpha > \beta$ by Appendix I as in the goodness of fit tests is statistically feasible and acceptable.

As for the range of correlation coefficient of LSD, $\mathrm{dif}(\theta) = \theta_2 - \theta_1$, having a range of first 1.0 (uneducated guess) and then gradually dropping to a 0.5 does generally, if not always, have a subtle savings effect. This is why a Generalized Beta prior [5] was chosen to incorporate the expert opinion for the range of θ and recognize the infeasibility of very low imposed θ to lend freedom to versatility rather than assuming the default case of $\theta_2 - \theta_1 = 1.0$ flat when anything goes to avoid statistically unrealistic autocorrelation values of θ. Note that in Appendix I, the goodness of fit chi-square tests do not involve counts of zero for the underlying LSD tested as the r.v. w for LSD takes on nonzero values, $\mathrm{w} = 1, 2, 3\ldots$ as shown in (2) where the constant "a" is given by (3). Therefore, the blocks will show the frequencies of nonzero entities, where the zero count can be found by subtracting from the total number of test cases for each data set.

Screen 1 displays the menu of the aforementioned parameters, including the moving average with a default value of 1 and the goodness of fit test as in Appendix I. It has been found that for DR5, a moving average of 32 gives the best smoothing practice, which in turn calculates the corresponding exponential smoothing factor of 0.0606 [47]. Variable cost data can also be applied, either with a linearly changing slope or by using forced data of the cost parameters c, b, and a, respectively, for each test case entered.

V. DISCUSSION AND CONCLUSION

The contribution of proposed methodology lies in an empirical Bayesian approach to determine an economically efficient stopping rule in a CP setting that takes the accumulation of failure clumps at each step into account in a software-failure (or branch coverage) counting process. This work is a follow-up to previous research done on Poisson ^ LSD as applied to computer software testing [10]–[13]. This research also presents an alternative to the previous publication in that the compounding distribution was assumed to be geometric (hence, Poisson ^ Geometric) due to its forgetfulness or independence property of the clumped failures and where additionally the stochastic time index was assumed to be in terms of CPU seconds [6]. This paper also addresses the effort-domain problem where the unit tests per calendar weeks are now replaced by test cases or test vectors as sometimes called in embedded-chips testing. However, in this paper, the compounding density is an LSD where failures are interdependent assumed to affect each other adversely by employing test cases as opposed to a continuous time

TABLE II

STOPPING RULES "S(·) = s" OF $\mathbf{DR1}(\#\mathbf{NC} = \mathbf{134}$ IN $\#\mathbf{TC} = \mathbf{200})$ OF ROWS 1–4, $\mathbf{DR2}(\#\mathbf{NC} = \mathbf{92}$ IN $\#\mathbf{TC} = \mathbf{185})$ OF ROWS 5–8, $\mathbf{DR3}(\#\mathbf{NC} = \mathbf{44}$ IN $\#\mathbf{TC} = \mathbf{100})$ OF ROWS 9–12, $\mathbf{DR4}(\#\mathbf{NC} = \mathbf{63}$ IN $\#\mathbf{TC} = \mathbf{200})$ OF ROWS 13–16, $\mathbf{DR5}(\#\mathbf{NC} = \mathbf{46}$ IN $\#\mathbf{TC} = \mathbf{2176})$ OF ROWS 17–20 WHEN $\alpha = 8$ AND $\beta = 2$. (*). WITH THE TWO-STAGE APPLICATION, THERE IS ADDITIONAL +\$58 000 FOR DR5 AS CALCULATED IN TABLE VI. ALL OF TABLE II'S STOPPING RULES SHOWN ARE ONE-STAGE TESTING RESULTS FOR SIMPLICITY PURPOSES

#TC	CL (#MC)	θ_2 - θ_1=1.	θ_2 - θ_1=.5	(1)$a \le$	(2)$a \le$	(3)b≥	(4)c≥	(5)Saved	(6)Lost
DR1									
NA	NA	S(4)=38	S(2)=36						
100	NA	S(100)=94	S(100)=94						
200	0.8 (107)	S(126)=108	S(125)=108	\$1284	\$6692	\$19972	\$1405	+\$11500	-\$252600
200	0.9 (121)	S(169)=132	S(167)=126	\$2550	\$32000	\$19845	\$258	+\$8500	-\$16900
DR2									
NA	NA	S(3)=23	S(2)=23						
93	NA	S(93)=52	S(93)=52						
185	0.8 (74)	S(131)=74	S(130)=74	\$1300	\$7000	\$19970	\$1333	+\$9500	-\$174600
185	0.9 (83)	S(154)=90	S(152)=84	\$2550	\$32000	\$19845	\$258	+\$8500	-\$16900
DR3									
NA	NA	S(5)=4	S(5)=4						
49	NA	S(49)=27	S(49)=27						
100	0.8 (35)	S(70)=36	S(69)=36	\$1375	\$8500	\$19963	\$1067	+\$7500	-\$5000
100	0.9 (40)	S(81)=41	S(80)=41	\$1633	\$13667	\$19937	\$632	+\$7000	-\$1100
DR4									
NA	NA	S(4)=19	S(4)=19						
100	NA	S(101)=54	S(101)=54						
200	0.8 (50)	S(95)=51	S(95)=51	\$1875	\$18500	\$19913	\$457	+\$41000	-\$109500
200	0.9 (57)	S(172)=57	S(171)=57	\$1466	\$10333	\$19953	\$857	+\$8500	-\$57200
DR5									
NA	NA	S(2)=4	S(2)=4						
1094	NA	S(1094)=40	S(1094)=40						
2176	0.8 (37)	S(100)=38	S(100)=38	\$26950	\$520000	\$17405	\$15	+\$1030000*	\$127700
2176	0.9 (41)	S(2042)=42	S(2042)=42	\$4350	\$68000	\$19665	\$119	+63000	-\$26600

TABLE III

STOPPING RULES USED IN CASE STUDY [49]

Orig	Original (without stopping rule)
SS1	Sequential Sampling Fixed
SS2	Sequential Sampling Variable
HW1	Howden First Formula
HW2	Howden Second Formula
BM	Binary Markov Model
DL	Dalal-Mallows Model
CP	Compound Poisson Rule
SB	Static Bayesian Rule
DB	Dynamic Bayesian Rule
CDB	Confidence-Based Dynamic Bayesian Rule

domain in terms of CPU seconds or hours or weeks. Recall that the dual of a time-dependent Poisson process is a time-independent discrete Bernoulli process whose theory is sufficiently strong to handle the unit test-case phenomenon replacing the unit test-week as a stochastic index where the response variable is the number of failures or branch coverage etc. [20]. This is in line with the test-case based testing activity, where limiting distribution of the sum of the nonhomogeneous Bernoulli variables is the Poisson(λ) process, where $\lambda t = np$, with n = number of Bernoulli trials and p = probability of detecting a failure or covering a branch ([9] and [50], p. 304).

The stopping rule is applied to five effort-domain test data sets, namely DR1 to DR5 compiled by CS Labs at Colorado State University [17]–[19], [48], [49] and also to a DoD data set [41], [44]. This stopping-rule method is a new derivative of the original publications on "The Compound Poisson Reliability Model" [11], [12]. The number of failures or branches covered is independent from test-case to test-case. Test cases are randomized, and not in any specific order. However, the total number of contributions or coverage at each one-step-look-ahead check assures the testing activity to stop due to a specified criterion "d" for a set of specified cost parameters α and β imposed on the data set itself having learned from previous similar activity or subjective guesswork. Then, the software analyst can apply a subsequent testing strategy after stopping due to saturation effect with respect to an economic criterion, provided that a desired confidence level is satisfied in order to see the effect of the following scheme.

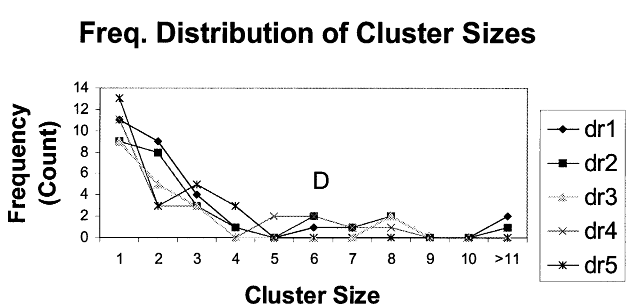
Therefore, the same algorithm can be applied for a next strategy to judge where to stop. Hence a mixed sequence of strategies can be employed for best efficiency to save time and effort, i.e., overall resources. This is sometimes called "mixed strategy testing" [17]–[19], [41], [44]. It is shown by McDaid and Wilson [38] that two stage sampling is superior to single stage as illustrated in our examples given in Screen 1 and Graphs 1 and 2. It is most likely that by sacrificing only small percentage of failure or branch coverage accuracy, one can literally spare wasted testing resources because of persisting on the same futile testing strategy, on a journey to the unknown. Also, as "d" gets smaller, usually stopping is delayed for fine-tuning. The saving of testing resources can be considerably crucial for large testing problems. This stopping-rule method is therefore based on a Bayesian approach of updating the historical information experienced for future decision-making. It assumes by Poisson ^ LSD (negative binomial, for a special case) model where the contributed failure or branch coverage clumped in a test-case is positively correlated. This implies that an occurrence of failure or branch is likely to invite a next failure or branch. For further research, a variety of informative priors can be considered as alternatives to conjugate prior generalized Beta (α, β) for θ [5], [21].

Further, to provide readers with fundamental information about what sorts of methods currently exist for a variety of projects as listed in Table III for the stopping-rule problem this paper is dealing with, and to provide evidence that the method proposed herein is a substantial improvement, lists of comparisons over the other existing methods in circulation are presented in Appendix II. In summary, the proposed MESAT is progressive and more data friendly in terms of its EDA that other methods do not attempt to study for diagnosis. MESAT is suitable for those data sets, which satisfy the goodness of fit criterion of their clump-size distribution with respect to a hypothesized LSD. This property of MESAT is therefore discriminative, rather than fitting for all purposes. It is generally true that the branch coverage data sets obey the assumptions stated at least in this paper. This is why five out of five data sets proved positive for the LSD assumed; hence good fits are declared for NBD in natural consequence by (1) to (13) in Section II. MESATs only seemingly subtle disadvantage is the assumption of independent (randomized) test cases, which is a requirement for the independent increments property of the Poisson processes as the major underlying distribution of counts in this research. However, as earlier explained in Section I, the randomization assumption is a practical reality in testing practice. Even if otherwise suspected, there is no universally accepted solution of modeling the correlation of test cases for each testing activity whose results are not known in advance by the nature of surprise factor in software testing.

APPENDIX I

DIAGNOSTIC CHECKS FOR EXPERIMENTAL DATA SETS

ClusterSize	Dr1	Dr2	Dr3	Dr4	Dr5
1	11	9	9	11	13
2	9	8	5	3	3
3	4	3	3	3	5
4	1	1	0	0	3
5	0	0	0	2	0
6	1	2	0	2	0
7	1	1	0	1	0
8	2	2	2	1	0
9	0	0	0	0	0
10	0	0	0	0	0
>11	2	1	0	0	0



Decision	GoodFIT		alpha=	0.05
	Pvalue=	0.14946	n=	32
a=	0.62133	theta=	0.8	
X=	P=	E=	O=	ChiSq=
1	0.497064	15.90605	11	1.513217
2	0.198826	6.362419	9	1.093426
3	0.10604	3.39329	4	0.108478
4	0.063624	2.035974	1	0.52714
5	0.040719	1.303023	0	1.303023
6	0.027146	0.868682	1	0.019851
7	0.018615	0.595668	1	0.274456
8	0.01303	0.416968	2	6.010041
9	0.009266	0.29651	0	0.29651
10	0.006671	0.213487	0	0.213487
11	0.018998	0.60793	2	3.187638
	DataSet=	**dr1**	ChiSqTot	14.54727

Decision	GoodFIT		alpha=	0.05
	Pvalue=	0.116527	n=	27
a=	0.62133	theta=	0.8	
X=	P=	E=	O=	ChiSq=
1	0.497064	13.42073	9	1.456168
2	0.198826	5.368291	8	1.290148
3	0.10604	2.863089	3	0.006547
4	0.063624	1.717853	1	0.299975
5	0.040719	1.099426	0	1.099426
6	0.027146	0.732951	2	2.190344
7	0.018615	0.502595	1	0.492269
8	0.01303	0.351816	2	7.721385
9	0.009266	0.250181	0	0.250181
10	0.006671	0.18013	0	0.18013
11	0.018998	0.512941	1	0.462484
	DataSet=	**dr2**	ChiSqTot	15.44906

Decision	GoodFIT		alpha=	0.05
	Pvalue=	0.078483	n=	19
a=	0.62133	theta=	0.8	
X=	P=	E=	O=	ChiSq=
1	0.497064	9.444216	9	0.020894
2	0.198826	3.777686	5	0.395494
3	0.10604	2.014766	3	0.481786
4	0.063624	1.20886	0	1.20886
5	0.040719	0.77367	0	0.77367
6	0.027146	0.51578	0	0.51578
7	0.018615	0.353678	0	0.353678
8	0.01303	0.247574	2	12.40433
9	0.009266	0.176053	0	0.176053
10	0.006671	0.126758	0	0.126758
11	0.018998	0.360958	0	0.360958
	DataSet=	**dr3**	ChiSqTot	16.81826

Decision	GoodFIT		alpha=	0.05
	Pvalue=	0.476983	n=	23
a=	0.62133	theta=	0.8	
X=	P=	E=	O=	ChiSq=
1	0.497064	11.43247	11	0.01636
2	0.198826	4.572989	3	0.541067
3	0.10604	2.438927	3	0.129074
4	0.063624	1.463356	0	1.463356
5	0.040719	0.936548	2	1.207551
6	0.027146	0.624365	2	3.03087
7	0.018615	0.428136	1	0.763841
8	0.01303	0.299695	1	1.636417
9	0.009266	0.213117	0	0.213117
10	0.006671	0.153444	0	0.153444
11	0.018998	0.436949	0	0.436949
	DataSet=	**dr4**	ChiSqTot	9.592047

Decision	**GoodFIT**		alpha=	0.05
	Pvalue=	0.651303	n=	24
a=	0.62133	theta=		**0.8**
X=	P=	E=	O=	ChiSq=
1	0.497064	11.92954	13	0.096055
2	0.198826	4.771814	3	0.657889
3	0.10604	2.544968	5	2.368275
4	0.063624	1.526981	3	1.420965
5	0.040719	0.977268	0	0.977268
6	0.027146	0.651512	0	0.651512
7	0.018615	0.446751	0	0.446751
8	0.01303	0.312726	0	0.312726
9	0.009266	0.222383	0	0.222383
10	0.006671	0.160116	0	0.160116
11	0.018998	0.455947	0	0.455947
	DataSet=	**dr5**	ChiSqTot	7.769886

APPENDIX II
COMPARISONS OF THE PROPOSED CP RULE WITH OTHER STOPPING RULES

Almost all of the existing statistical models used to determine stopping-points stem from research results in software engineering [22], [34], [37]–[41], [44]. Many models have been proposed assessing the reliability measurements of software systems to help designers evaluate, predict, and improve the quality of their software systems [23]–[32]. However, software reliability models aim at estimating the remaining faults in a given software program, which makes the direct use of such models not beneficial in estimating the number of remaining uncovered branches in a behavioral model since the remaining uncovered branches are known. Instead, the estimation process can be slightly modified to focus on the expected number of faults, or coverage items in the case of behavioral model verification, within the next unit of testing time. Unfortunately, all the existing software reliability models assume that failures occur one at a time, except for the proposed MESAT approach that uses a CP. Based on this assumption, expectations of the times between failures are carried on. In observing new coverage items in a behavioral model, branches are typically covered in clumps. Besides, in the proposed MESAT tool, the positive correlation within a clump is taken into account.

The confidence-based modeling approach [27], [28] takes advantage of hypothesis testing in determining the saturation of the software failure. A null hypothesis H_0 is performed and later examined experimentally based on an assumed probability distribution for the number of failures in a given software. Suppose that a failure has a probability less than or equal to B to occur, then we are at least $1 - \mathsf{B}$ confident that H_0 is true. Similarly, if the failures for the next period of testing time have the same probability of at least B to occur, then for the next N testing cycles, we have a confidence of at least C that no failures will happen, where

$$\mathsf{C} = 1 - (1 - \mathsf{B})^{\mathrm{N}} \tag{A2.1}$$

$$\mathrm{N} = \frac{\ln(1 - C)}{\ln(1 - B)}. \tag{A2.2}$$

If $\mathrm{C} = 0.95, \mathrm{B} = 0.3$, then by using (A2.2), $\mathrm{N} \approx 100$. This is a single-equation stopping-rule method, which can be likened to a parallel system of N independent components whose reliabilities are identical to be each $\mathrm{R} = \mathrm{B}$ to satisfy an overall network reliability of C ([36], p. 265). To apply Howden's model to the process of HDL verification, we first need to create failures as interruptions, where an interruption is an incident where one or more new part of the model are exercised. Using branch coverage as a test criterion, an interruption, therefore indicates one or more new branches are covered. We set a probability for the interruption rate B and choose an upper-bound level of confidence C. Experimentally, we do not examine the hypothesis unless the interruption rate becomes smaller than the preset value B. When so, we calculate the number of test patterns needed to have at least C confidence of not having any new branch in the next N patterns and run them. If an interruption occurs, we continue examining the hypothesis until we prove it and then stop. In this approach, we assume that coverage items, or indeed interruptions are independent and have equal probabilities of being covered. The rate of interruption is decreasing and we assume no interruptions will occur in the next N test cases; then, the expected probability of interruptions will be [27], [28], [34]

$$\mathrm{B_t} = \frac{B}{t + T} \tag{A2.3}$$

where T is the last checked point in testing, and this leads to the reformulation of (A2.1) as follows:

$$\mathsf{C} = 1 - \prod_{t=1}^{N} \left(1 - \frac{B}{t + T}\right)^{\mathsf{N}}. \tag{A2.4}$$

Branches in behavioral models are usually covered in clumps during the verification process. One could consider the event of having one or more new covered branches as an interruption. Thus, interruptions could be treated as failures with a chosen upper-bound probability B where the hypothesis is not examined unless the interruption rate becomes smaller than this preset value B. The confidence of having an interruption is then calculated based on the interruption rate. Nevertheless, it was assumed that interruptions are independent of each other. Some authors support that this is not correct [33], [34]. In fact, branches in behavioral models can be classified as dominant, and controlled branches where it is impossible to cover the lower level branches without covering their dominant branches. Moreover, the sizes of the interruptions are not modeled in [27], [28] making the understanding of the branch behavior less informative. Sanping Chen and Shirley Mills developed a statistical Markov process or Binary Markov model [29], [34] where the probabilistic distribution assumptions are the same as confidence-based model except that failures are statistically dependent with a certain unknown correlation constant, ρ. Again, if interruptions are correlated, then the probability of having no interruptions in the next N test cases is

$$\mathsf{p}(0 \,|\, \mathsf{N}; \mathsf{B}; \rho) = (1 - \mathsf{B})(1 - \mathsf{B} + \rho \cdot \mathrm{B})^{\mathsf{N}-1} \tag{A2.5}$$

that makes the confidence as $\mathsf{C} = 1 - \mathsf{p}(0 \,|\, \mathsf{N}; \mathsf{B}; \rho)$. Chen and Mill's model still doesn't deal with the issue of clumping. Furthermore, the value of ρ in this model is unknown, and authors experimentally assumed different values ranging from 0 to 0.9 and obtained different results. Thus, ρ needs to be determined experimentally.

In Howden's model, the assumption that failures or interruptions have a given probability B independently is erroneous.

TABLE IV
RESULTS OF STOPPING-RULE COVERAGE FOR STATIC CASE STUDY WHERE COVERAGE PER TESTING PATTERN CAN BE CALCULATED (i.e., COVERAGE/PATTERNS) WITHOUT USING COST FACTORS [49]

Model	Orig	SS1	SS2	HW1	HW2	BM	DL	CP	SB	DB	CDB
Sys7	568	536	538	536	536	536	536	547	535	536	535
	5428	1039	1858	927	969	1025	1235	6287	661	563	569
8251	161	73	73	79	79	81	75	112	74	73	67
	8150	3259	3812	2769	2906	3033	5712	9600	2275	2239	2091
B01	200	177	142	128	155	128	128	135	128	128	128
	8000	8169	3352	1010	1108	1211	1211	4200	1854	914	897
B04	223	220	218	206	214	214	219	217	199	202	202
	8000	1028	5047	1894	2468	2557	1175	1710	631	674	742
B05	259	234	251	251	251	251	252	253	232	233	233
	8000	1079	7122	2092	2343	2431	5318	1080	808	745	744
B06	210	192	192	192	192	192	204	204	192	192	192
	8000	8725	4618	1240	1407	1439	7110	4500	673	708	708
B07	210	196	198	196	204	196	196	204	195	195	195
	8000	8963	4660	1322	3904	1621	1132	4500	789	704	731
B08	274	268	268	263	263	273	273	273	273	273	273
	8000	1244	6122	1392	1405	2283	8427	9600	2249	2033	1829
B09	260	234	251	234	251	251	252	253	232	233	233
	8000	1079	7122	1512	2053	2470	5324	7800	809	734	735
B10	210	197	198	204	204	204	196	208	208	208	208
	8000	9068	4660	1488	1711	1781	915	4200	2181	1488	1240
B11	223	220	218	206	214	214	219	217	199	202	202
	8000	1028	5047	1894	2468	2557	1175	1710	631	674	742
B12	259	234	251	234	251	251	252	253	232	233	233
	8000	1079	7122	1545	2085	2462	5318	6900	808	745	744
B14	257	248	248	244	244	244	248	253	245	245	245
	8000	1136	5712	1892	1900	1991	1618	2100	1982	735	748
B15	415	351	351	350	350	350	418	383	364	364	364
	8000	1619	7906	1892	1900	1991	8000	9000	2080	2298	2010

Branches in an HDL model, as we know are strongly dependent of each other. In fact, we can classify some branches where it is impossible to cover the lower level ones without covering their dominants. Moreover, the clump sizes caused by the interruptions are not modeled in this study making the decision of continuing or stopping the testing process inaccurate. Last, this work does not incorporate the cost of testing or releasing the product, and the goal of testing in the first place is not only having a high quality product but also minimizing the testing costs [34].

Dallal and Mallows [1] assumed that the total number of failures in a given software is a random variable with unknown mean, and the number of failures occur during the testing time is a nonhomogeneous Poisson process with increments $\lambda g(t)$. The time needed for a single failure to occur is distributed as $g(t)$, which can be assumed exponential. This model has a better description for the failure process over the previous models so far discussed, such as Howden and modified Howden's in Chen and Mill's models. However, it still suffers from the problem of not having more than one interruption at a time, which reduces the efficiency of the model when applying it to branch coverage estimation [34].

Finally, the author of this manuscript, Şahinoğlu *et al.* [11]–[13], [17]–[19] whereas applied a CP model that models the branch coverage process of VHDL circuits utilizing the benefits of the "Dallal and Mallows" economic model by reformulating it [6] and solving the clumping phenomenon of branches being covered in the testing process. This model uses the empirical Bayesian principles for the compounded Poisson counting process. It was previously introduced as a software reliability model for the remaining number of failures' estimation in 1992 [11] and later modified to incorporate a version of the cost modeling proposed by Dallal and Mallows in 1995 [6], [13]. Recently, it was formulated to model the branch coverage process in behavioral models [17]–[19]. The idea is to compound potentially two probability distributions, for both the number of interruptions and the size of interruptions. The resulting compound distribution is assumed to be the probability distribution function of the total number of failures, or coverage items, at a certain testing time point. The parameters of the distributions are also assumed to be random variables based on the empirical Bayesian estimation. For modeling the branch coverage process for behavioral models, it is assumed that the number of interruptions over the time, $N(t)$, is a Poisson process with mean λ, and the size of each given interruption, w_i, is distributed as a Logarithmic Series Distribution (LSD). See diagnostics of Appendix I for the justification of LSD of clump sizes. The resulting compound distribution for the total number of failures, which is the sum of the sizes, is also known as an NBD, if the Poisson parameter λ is set to $-k\ln(1-\theta)$. The CP model takes into account the clumps of the coverage items in a statistical manner by updating the assumed probability distribution parameters in every test-case based on the testing history. However, interruptions in the testing process are assumed to be independent, mainly due to the "independent increments property" of the anchoring Poisson process. The proposed MESAT also incorporates a minimal confidence rule

TABLE V
RESULTS OF COST ANALYSIS FOR A DYNAMIC CASE STUDY WHERE $a = \$5000$, $b = \$500$, AND $c = \$1$ [44]

Model	< higher		Rank by savings/benefit				lower >			
Sys7:	**CP**	SS2	DB	HW1	HW2	BM	SS1	DL	CDB	SB
	-46504	-82575	-90280	-90644	-90686	-90742	-90756	-90952	-94786	-94878
825I:	**CP**	BM	HW1	HW2	DL	SB	DB	SS1	SS2	CDB
	-148600	-281533	-290269	-290406	-311212	-312275	-316739	-317759	-318312	-343591
B01:	SS1	HW2	SS2	**CP**	CDB	DB	HW1	BM	DL	SB
	-31669	-133584	-184352	-216700	-244897	-244914	-245010	-245211	-245211	-245854
B04:	SS1	SS2	DL	HW2	BM	**CP**	HW1	DB	CDB	SB
	56218	52453	50245	37032	36943	35900	1606	-15174	-15242	-28631
B05:	DL	**CP**	HW1	HW2	BM	SS2	CDB	DB	SB	SS1
	43182	42200	41908	41657	41569	36878	-37744	-37745	-42308	-43295
B06:	**CP**	DL	SB	DB	CDB	HW1	HW2	BM	SS2	SS1
	48500	45890	-1673	-1708	-1708	-2240	-2407	-2439	-5618	-9725
B07:	HW2	**CP**	SS2	DL	HW1	BM	DB	CDB	SB	SS1
	49096	48500	21340	15868	15678	15379	11796	11769	11711	8037
B08:	CDB	DB	SB	BM	DL	**CP**	SS2	SS1	HW1	HW2
	73671	73467	73251	73217	67073	65900	46878	40553	29108	29095
B09:	**CP**	DL	HW2	BM	SS2	HW1	DB	CDB	SB	SS1
	40700	38676	37447	37030	32378	-38512	-42234	-42235	-46809	-47795
B10:	CDB	DB	SB	**CP**	HW1	HW2	BM	SS2	DL	SS1
	69760	69512	68819	66800	51512	51289	51219	21340	16085	12432
B11:	SS1	SS2	DL	HW2	BM	**CP**	HW1	DB	CDB	SB
	56218	52453	50245	37032	36943	35900	1606	-15174	-15242	-28631
B12:	**CP**	DL	HW2	BM	SS2	HW1	CDB	DB	SB	SS1
	46100	43182	41915	41538	36878	-34045	-37744	-37745	-42308	-43295
B14:	**CP**	DL	SS2	SS1	DB	CDB	SB	HW1	HW2	BM
	41000	37882	33788	28133	25265	25252	24018	19608	19600	19509
B15:	DL	**CP**	CDB	SB	DB	HW1	HW2	BM	SS2	SS1
	13498	-73000	-151510	-151580	-151798	-214392	-214400	-214491	-215906	-224190

in addition to applying the one-step-ahead formula of (27) for assessing whether to stop or continue economically.

All the previously discussed stopping rules assume that the failures or interruptions are random processes according to a given probability distribution. A sequential sampling technique that doesn't involve any assumptions on the probability distributions for the failure process was presented in [25]. Recently, the technique is applied to VHDL models in determining the stopping points for a given testing history of branch coverage [30]. The model evaluates the stopping decision based on three key factors: the discrimination ratio (τ), the supplier risk (α), and the consumer risk (β). If the number of cumulative coverage at time t is $\mathsf{X(t)}$, then the testing process should be stopped at

$$\mathsf{X(t)} = \frac{\ln\left(\frac{1-\beta}{\alpha}\right) - \ln(\gamma)}{1-\gamma}. \tag{A2.6}$$

The stopping decision strongly depends on the value of γ much more than α and β. The decision doesn't incorporate any cost model of the testing process. In [25], the variable was modified with respect to testing strategies such that if higher coverage were achieved in the previous test strategy, the value of is increased in the current test strategy in order to decrease the expectation of achieving more coverage in the current strategy. The new value of γ, therefore, becomes: $\gamma + = \gamma \ln(\Delta)$, where Δ is the coverage increase achieved in the previous test strategy The value of γ, however, remains the same if $\Delta < \mathrm{e}$. This type of statistical modeling doesn't use any priori probability distribution for the data provided. This is one reason why the sequential sampling models are widely used in many testing areas [33], [34]. However, the cost of testing is not modeled in making the stopping decision. Moreover in the opinion of this manuscript's author, the stopping point determined by the sequential sampling model is very sensitive to the value chosen during the testing process. (A2.6) is an equation subject to an abusive use for purposes of experimental validation. Authors of this approach [30] have earlier suggested values for γ up to 250, whereas Musa's [25] text only uses γ in the order of 5 or 10. Excessive values of γ pose a contradiction and threat to the Wald's SPRT theory for sequential testing in terms of type I (whose probability is α) and II (whose probability is β) errors. The same holds true for α which authors in their related paper have suggested to be $\alpha = 0.50$, a relatively exaggerated value compared to Musa's $\alpha = 0.10$. Singpurwalla *et al.* [37], [40], [45], McDaid and Wilson [38] and Ross [39] have developed their own stopping rules with differing statistical assumptions on one- or two-stage testing schemes. However, because these techniques have not been experimented on "hardware or silicon testing" with respect

TABLE VI
RESULTS OF DR5 MIXED STRATEGY STOPPING- RULE AT A MINIMUM 80% CONFIDENCE LEVEL

Week	Lambda	k	w	X	E(X)	e(X)	Percentage
1	1.0	0.57711	4	4	4.391	N / A	8.7
5	0.4	0.20036	2	6	2.337	0.936	13.04
6	0.5	0.2266	4	10	3.325	0.988	21.74
7	0.57143	0.24445	3	13	4.125	0.8	28.26
8	0.625	0.26348	1	14	4.588	0.463	30.43
9	0.66667	0.26808	3	17	5.285	0.697	36.96
10	0.7	0.27024	3	20	5.957	0.672	43.48
34	0.23529	0.08512	3	23	2.432	0.372	50.0
35	0.25714	0.0894	4	27	2.872	0.44	58.7
43	0.23256	0.07859	3	30	2.769	0.348	65.22
44	0.25	0.08385	1	31	3.017	0.248	67.39
52	0.23077	0.0767	1	32	2.849	0.223	69.57
66	0.19697	0.06435	2	34	2.539	0.232	73.91
76	0.18421	0.0597	1	35	2.423	0.174	76.09
91	0.16484	0.05299	1	36	2.214	0.153	78.26
99	0.16162	0.05124	2	38	2.242	0.18	82.61
100	0.16	0.05073	0	38	2.221	-0.022	82.61

```
Stop at X(100) = 38.0
Coverage = 82.6086956521739 %

Cost Analysis:
Cost of correcting all 46 errors by exhaustive - testing would have been
46000.00$
Cost of correcting 38 pre - release errors using MESAT is 38000.00$
Savings for not correcting the remaining 8 by using MESAT is 8000.00$

Cost of executing all 2176 test cases by exhaustive - testing would have
been 1088000.00$
Cost of executing 100 test cases by using MESAT is 50000.00$
Savings for not executing the remaining ( 2176 - 100 ) = 2076 test cases
is 1038000.00$

Results of using MESAT are:
Savings for not correcting the remaining 8 errors by using MESAT is
8000.00$
Plus the 1038000.00$ saved for not executing the remaining 2076 test cases
equals a total savings of 1046000.00$
Minus the 16000.00$ post - release cost of correcting 8 errors not covered
( 8 x 2000.00$ )
Total savings for using MESAT is 1030000.00$
```

strategy: 1
    Stop at X(100) = 38.0
    Coverage = 82.0%
    Total Coverage = 82.0 %
    Total Covered = 38
strategy: 2
    Stop at X(1959) = 7.0
    Coverage = 88.0%
    Total Coverage = 97.0 %
    Total Covered = 45
Strategy 1 Cost Analysis Summary: Total savings for using MESAT is 1030000.00$
Strategy 2 Cost Analysis Summary: Total savings for using MESAT is 58000.00$

Insufficient data for Strategy Number: 3

to branch coverage, no comparative results are available in the engineering literature in terms of merits.

The above arguments suggest that the proposed MESAT employing both a minimal confidence rule and one-step-look-ahead formula within a single or multistage testing scenario to justify a decision taken whether to continue or stop testing, has the imminent advantages of recognizing the clumping effect in coverage testing as well as incorporating the economic criteria in addition to its data discriminative traits by conducting EDA through diagnostic checks. It is imperative that a diagnostic check, such as in Appendix I, be undertaken if similar exhaustive test results are available. This is necessary to justify the usage of the LSD model for the clump sizes, a model that eventually leads to the NBD assumption for the total number of coverage by default in the wake of the expression $\lambda = -k\ln(1-\theta) = k\ln q$ assumed to hold true.

For a more thorough comparative case study, research done by Hajjar and Chen was utilized [22], [49], where nine different stopping rules, shown in Table III were applied to 14 different VHDL models [44]. The results of the stopping-rule determinations are shown in Table IV, including results without the use of any stopping rule. This stopping-rule comparison portrays the CP method as having one of the lowest efficiencies based on a naive "coverage per testing pattern" index, which is defined as the number of branches covered divided by the total test patterns used. Despite their index rating, CP found the most faults for 10 out of the 14 VHDL models, while ranking second in B15, third in B01 and fourth in B04. Furthermore, no economic analysis has been performed to illustrate the monetary gain or loss associated with the various stopping rules. Let us now use the cost benefit criterion of (31) in the main paper, where RF is the remaining number of failures uncovered and RT is the remaining

TABLE VII
RESULTS OF DR5 MIXED STRATEGY STOPPING RULE AT A MINIMUM 90% CONFIDENCE LEVEL

```
strategy: 1
        Stop at X(2042) = 42.0
        Coverage = 91.0%
        Total Coverage = 91.0 %
        Total Covered = 42
strategy: 2
        Stop at X(86) = 4.0
        Coverage = 100.0%
        Total Coverage = 100.0 %
        Total Covered = 46
Strategy 1 Cost Analysis Summary: Total savings for using MESAT is 63000.00$
Strategy 2 Cost Analysis Summary: Total savings for using MESAT is 0$
```

number of test patterns unused when stopped. For an example, we will use $\mathrm{c} = \$1, \mathrm{b} = \$230, \mathrm{a} = \$2300$ since cost of redemption of after-market is 10 times more than that of before. Using the Sys7 data with CP, we get [44], [48]:

$\$2300(568 - 547) = \$2300(21) = \$48\,300 < \$230(21) + \$1(54283 - 6287) = \$52\,826$, showing CP to be cost effective by $\$52\,826 - \$48\,300 = \$4526$. Comparing Sys7 with DB, we get $\$2300(568 - 536) = \$2300(32) = \$73\,600 < \$230(32) + \$1(54283 - 563) = \$61\,080$, showing DB to not be cost effective by $\$61\,080 - \$73\,600 = -\$12\,520$. Why a ratio of 10 used between before-release and after-release costs? The main reason is that silicon testing, unlike software testing, is more expensive for uncovered branches or failures. Tables VI and VII illustrate the results of a mixed-strategy testing activity.

Although access to the VHDL model data used in Hajjar *et al.*'s research [49] was not available, a cost analysis could still be applied to their results. By performing this cost analysis on the stopping points in Table III and comparing the results, the economically beneficial stopping rules were determined for a given cost criterion. Using the cost criterion of (31), in a case study in which a cost index was applied to the data with cost values of $\mathrm{a} = \$5000, \mathrm{b} = \$500$, and $\mathrm{c} = \$1$. The CP stopping rule was clearly more beneficial. As can be seen in Table V, of the nine stopping rules used in that study, the CP stopping rule ranked very high with regards to savings in many of the VHDL data sets used for comparison. Low cost of testing in conjunction with high post-release repair cost render the CP stopping rule superior to many of the other stopping rules in the study. The incentive behind the mixed strategy testing is that a bug undetected in a silicon embedded chip is much more costly than a bug in software, and, therefore, the stopping rule needs to be very conservative. At the end of the spectrum, because the cost of testing is much less than the cost of a bug in silicon, it seems that a nonconservative stopping rule is worse than some other rules. Another angle can be extracted from the Table IV, where the number of branch coverage in SB (Hajjar and Chen's proposed rule) is more than 10% $*$fewer$*$ than the original (no stopping rule). This is probably not acceptable in hardware. A comparison: ATPG typically aims for 90+% fault coverage, and a user would probably aim for even 1% increases in coverage point, if it is achievable in a reasonable amount of computation. So the proposed rule is probably a good rule to "switch" instead of "stop" the testing process.

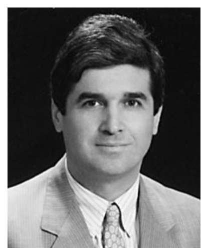
ACKNOWLEDGMENT

The author would like to thank Prof. A. S. Al-Khalidi, from Texas A&M University's Institute of Statistics and formerly a Senior Faculty Member at Dokuz Eylul University's Department of Statistics, Izmir, Turkey, for his earlier theoretical contributions as cited. The author would also like thank J. Larson and M. Hicks for expert java programming, as well as the two distinguished Guest Editors of the Special Section on Testing of the IEEE Instrumentation and Measurement Society, for their time and effort, and the Instrumentation and Measurement Transactions Administrator for his assistance.

REFERENCES

[1] S. R. Dallal and C. L. Mallows, "When should one stop testing software," *J. Amer. Stat. Assoc.*, vol. 83, pp. 872–679, 1988.
[2] M. H. DeGroot, *Optimal Statistical Decisions*. New York: Mc Graw-Hill, 1970.
[3] N. Johnson, S. Kotz, and J. Kemp, *Univariate Discrete Distributions*, 2nd ed. New York: Wiley, 1993.
[4] S. Kotz *et al.*, *Encyclopedia Stat. Sciences*, 1988. Vol. 5, 11–113; Vol. 6, 169–176.
[5] T. G. Pham and N. Turkkan, "Bayes binomial sampling by attributes with a generalized-beta prior distribution," *IEEE Trans. Rel.*, vol. 41, pp. 310–316, Mar. 1992.
[6] P. Randolph and M. Şahinoğlu, "A stopping-rule for a compound Poisson random variable," *Appl. Stochastic Models Data Anal.*, vol. 11, pp. 135–143, June 1995.
[7] S. Samuels, *Handbook of Sequential Analysis*, B. K. Ghosh and P. K. Sen, Eds. New York: Marcel Dekker, 1991, ch. 16, pp. 381–405.
[8] C. C. Sherbrooke, *Discrete Compound Poisson Processes and Tables of the Geometric Poisson Distribution*. Santa Monica, CA: Memorandum RM-4831-PR, The Rand Cooperation, July 1966.
[9] M. Şahinoğlu, "The limit of sum of Markov Bernoulli variables in system reliability evaluation," *IEEE Trans. Rel.*, vol. 39, pp. 46–50, Apr. 1990.
[10] ——, "Negative binomial density of the software failure count," in *Proc. 5th Int. Symp. Computer and Information Sciences (ISCIS)*, vol. 1, Oct. 1990, pp. 231–239.
[11] ——, "Compound Poisson software reliability model," *IEEE Trans. Software Eng.*, vol. 18, pp. 624–630, July 1992.
[12] M. Şahinoğlu and U. Can, "Alternative parameter estimation methods for the compound Poisson software reliability model with clustered failure data," *Software Testing Verification Rel.*, vol. 7, pp. 35–57, Mar. 1997.
[13] M. Şahinoğlu and A. S. Al-Khalidi, "A Bayesian stopping-rule for software reliability," in *Proc. 5th World Meeting ISBA, Satellite Meeting to ISI-1997*, Aug. 1997.
[14] M. Şahinoğlu, J. J. Deely, and S. Capar, "Stochastic Bayes measures to compare forecast accuracy of software-reliability models," *IEEE Trans. Rel.*, pp. 92–97, Mar. 2001.
[15] N. Johnson, S. Kotz, and N. Balakrishnan, *Continuous Univariate Distributions*, 2nd ed., 1995, vol. 2.
[16] G. G. Roussas, *A First Course in Mathematical Statistics*. Reading, MA: Addison-Wesley, 1973, p. 253.
[17] T. Chen, M. Şahinoğlu, A. von Mayrhauser, A. Hajjar, and C. Anderson, "How much testing is enough? Applying stopping-rules to behavioral model testing," in *Proc. 4th Int. High-Assurance Systems Engineering Symposium (HASE)*, Nov. 17–19, 1999, pp. 249–256.
[18] M. Şahinoğlu, A. von Mayrhauser, A. Hajjar, T. Chen, and Ch. Anderson, "On the efficiency of a compound Poisson stopping-rule for mixed strategy testing," in *Proc. IEEE Aerospace Conf.*, Aspen, CO, Mar. 6–13, 1999.
[19] T. Chen, M. Şahinoğlu, A. von Mayrhauser, A. Hajjar, and C. Anderson, "Achieving the quality of verification for behavioral models with minimum effort," in *Proc. 1st Int. Symp. Quality Electronic Design (ISQED)*, San Jose, CA, Mar. 20–22, 2000, pp. 234–239.
[20] E. Çınlar, *Introduction to Stochastic Processes*. Englewood Cliffs, NJ: Prentice-Hall, 1975.
[21] M. Şahinoğlu and A. Al-Khalidi, "A stopping-rule for time-domain software testing," in *Proc. 10th Int. Symp. Software Reliability Engineering*, Boca Raton, FL, Nov. 1–4, 1999.

[22] A. Hajjar and T. Chen, "A new stopping-rule for behavioral model verification based on statistical Bayesian technique," *IEEE Trans. Computer-Aided Design*, to be published.
[23] S. Gokhale and K. Trivedi, "Log-logistic software reliability growth model," *Proc. 3rd IEEE Int. High Assurance Systems Engineering Symp. (HASE)*, pp. 34–41, Nov. 1998.
[24] A. Goel, "Software reliability models: Assumptions, limitations, and applicability," *Software Eng.*, vol. SE-11, no. 12, pp. 1411–1423, Dec. 1985.
[25] J. Musa, "A theory of software reliability and its application," *Software Eng.*, vol. SE-1, no. 3, pp. 312–27, 1975.
[26] D. Mills, "On the statistical validation of computer programs," IBM FSD, Rep. FSC-72-6015, 1972.
[27] W. Howden, "Confidence-based reliability and statistical coverage estimation," *Proc. Int. Symp. Software Reliability Engineering*, pp. 283–291, Nov. 1997.
[28] ——, "Systems testing and statistical test data coverage," in *Proc. COMPSAC*, Aug. 1997, pp. 500–505.
[29] S. Chen and S. Mills, "A binary Markov process model for random testing," *Software Eng.*, vol. 22, no. 3, pp. 218–223, 1996.
[30] T. Chen, I. Munn, A. von Mayrhauser, and A. Hajjar, "Efficient verification of behavioral models using the sequential sampling technique," in *Proc. Symp. Very Large Scale Integration*, Brazil, 1999.
[31] B. Barrera, "Code coverage analysis-essential to a safe design," *Electron. Eng.*, pp. 41–44, Nov. 1998.
[32] B. Dickinson and S. Shaw, "Software techniques applied to VHDL design," *New Electron.*, no. 9, pp. 63–65, May 1995.
[33] A. Hajjar, T. Chen, and A. von Mayrhauser, "On statistical behavior of branch coverage in testing behavioral VHDL models," in *Proc. IEEE High Level Design Validation and Test Workshop*, Berkeley, CA, Nov. 2000.
[34] A. Hajjar, T. Chen, I. Munn, A. Andrews, and M. Bjorkman, "Stopping criteria comparison: Toward high quality behavioral verification," in *Proc. Int. Symp. Quality in Electronic Design*, San Jose, CA, Mar. 2001.
[35] G. G. Roussas, *A First Course in Mathematical Statistics*, 1st ed. Reading, MA: Addison-Wesley, 1973.
[36] E. E. Lewis, *Introduction to Reliability Engineering*, 2nd ed. New York: Wiley.
[37] E. H. Forman and N. D. Singpurwalla, "An empirical stopping rule for debugging and testing computer software," *J. Amer. Stat. Assoc.*, vol. 72, pp. 750–757, 1977.
[38] K. McDaid and S. P. Wilson, "Deciding how long to test software," *The Statistician*, vol. 50, pp. 117–134, 2001.
[39] S. M. Ross, "Software reliability: The stopping rule problem," *IEEE Trans. Software Eng.*, vol. 11, pp. 1472–1476, 1985.
[40] N. D. Singpurwalla, "Determining an optimal time interval for testing and debugging software," *IEEE Trans. Software Eng.*, vol. 17, pp. 313–319, 1991.
[41] M. Şahinoğlu, C. Bayrak, and T. Cummings, "High assurance software testing in business and DoD," *Trans. Soc. Design Process Sci.*, vol. 6, no. 2, pp. 107–114, 2002.
[42] K. Abdullah, J. Kimble, and L. White, "Correcting for unreliable regression integration testing," in *Proc. Int. Conf. Software Maintenance*, Nice, France, Oct. 1995, pp. 232–241.
[43] W. Notz, private communication, Aug. 2002.
[44] M. Hicks, "A Stopping Rule Tool for Software Testing," M.S. thesis, Dept. Comput. Inform. Sci., Troy State Univ., Montgomery, AL, Dec. 2000.
[45] N. D. Singpurwalla and S. P. Wilson, *Statistical Methods in Software Engineering*. New York: Springer, 1999.
[46] M. Şahinoğlu and E. H. Spafford, "A Bayes sequential statistical procedure for approving products in mutation-based software testing," in *Proc. IFIP Conf. Approving Software Products (ASP)*, Germany, Sept. 17–19, 1990, pp. 43–56.
[47] D. Anderson, D. J. Sweeney, and T. A. Williams, *An Introduction to Management Science—Quantitative Approaches to Decision Making*, pp. 735–743, 2002.
[48] M. Şahinoğlu and S. Glover, "Economic analysis of a stopping rule in branch coverage testing," in *Proc. 3rd Int. Symp. Quality Electronic Design*, San Jose, CA, Mar. 18–21, 2002, pp. 341–346.
[49] A. Hajjar and T. Chen, "Improving the efficiency and quality of simulation-based behavioral model verification using dynamic Bayesian criteriaxc," in *Proc. 3rd Int. Symp. Quality Electronic Design*, San Jose, CA, Mar. 18, 2002, pp. 304–309.
[50] K. S. Trivedi, *Probability and Statistics with Reliability, Queuing and Computer Science Applications*, 2nd ed. New York: Wiley, 2002.
[51] S. R. Das, C. V. Ramamoorthy, M. H. Assaf, E. M. Petriu, and W.-B. Jone, "Fault tolerance in sytems design in VSLI using data compression under constraints of failure probabilities," *IEEE Trans. Instrum. Meas.*, vol. 50, pp. 1725–1745, Dec. 2001.



Mehmet Şahinoğlu (S'78–M'81–SM'93) received the B.S. degree in electrical and computer engineering from the Middle East Technical University (METU), Ankara, Turkey, in 1973, the M.S. degree in electrical and computer engineering from the University of Manchester Institute of Science and Technology, Manchester, U.K., in 1975, and the Ph.D. degree in both electrical and computer engineering and statistics from Texas A&M University, College Station, in 1981.

Prior to joining the CIS Department, Troy State University, Montgomery, AL, as its first Eminent Scholar and Chairman in 1999, he was at METU for 20 years as a Reliability Consultant to the Turkish Electricity Authorities (TEK) and The National Defense Industry, Ankara, from 1976 to 1992, as a Professional Certified Engineer. He then served for five years as a Founder Dean of Science and Founder Chairman of the Department of Statistics at Dokuz Eylul University, Izmir, Turkey, from 1992 to 1997. He taught at Purdue University, West Lafayette, IN, from 1989 to 1990 and 1997 to 1998, and Case Western Reserve University, Cleveland, OH, from 1998 to 1999 as a Visiting Fullbright and NATO Research Scholar, respectively. He retired in 2000 after 26 years of civil service in Turkey as a Professor Emeritus. He published in electric power earlier in his career, and computer software reliability and testing in later years. He is accredited with the original findings of the "Compound Poisson Software Reliability Model" to account for the multiple (clumped) failures in predicting the total number of failures at the end of a mission time, and the "MESAT: Compound Poisson Stopping Rule Algorithm" in software testing literature. He is also jointly responsible, with Dr. David L. Libby, for the original derivation of the "Forced Outage Ratio (FOR)" or "Generalized Three-Parameter Beta (G3B)" pdf. He recently created an Exact Reliability Block Diagram Calculation (ERBDC) Tool, which is a novel graphical technique in the literature for quantifying and designing the reliability of computationally complex systems.

Dr. Şahinoğlu is a Fellow of the Austin-based Society of Design and Process Science and a member of ACM, AFCEA, ASA, and an elected member of ISI.